5-2.

ADD VELOCITIES w/ $\beta_T = +0.9c$

IN CM FRAME PIONS CAN GO BACK-TO-BACK

$$\pi^- \longleftarrow K^0 \longrightarrow \pi^+$$

CM FRAME

XFORMING TO LAB FRAME WILL PRODUCE PIONS w/ DIFFERENT SPEEDS

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2}$$

HERE, $u_x' = \pm 0.85c$ & $v = 0.9c$

$$u_x^{max} = \frac{(0.85 + 0.9)c}{1 + (0.85)(0.9)}$$

$$\boxed{u_x^{max} = 0.99c}$$

$$\boxed{u_x^{min} = \frac{(-0.85 + 0.9)c}{1 - (0.85)(0.9)} = 0.21c}$$

5-11

WANT RED LIGHT TO BE DOPPLER SHIFTED TO INFRARED

λ: 600 nm $\rightarrow$ 700 nm

$\Delta\lambda \sim$ 100 nm

DETERMINE NECESSARY β

$$\nu' = \left(\frac{1-\beta}{1+\beta}\right)^{1/2} \nu$$

OBJECTS TRAVEL AWAY FROM EACH OTHER

$$\lambda' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \lambda$$

$$\frac{\lambda' - \lambda}{\lambda} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} - 1$$

$$\frac{\Delta\lambda}{\lambda} = \frac{100\text{ nm}}{600\text{ nm}} = \frac{1}{6} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} - 1$$

$$\Rightarrow \frac{49}{36} = \frac{1+\beta}{1-\beta}$$

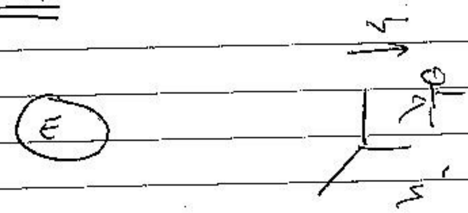
$$\alpha = \frac{1+\beta}{1-\beta} \qquad \text{w/ } \alpha = \frac{49}{36}$$

$$\alpha(1-\beta) = 1+\beta$$

$$\alpha - 1 = \beta(1+\alpha)$$

$$\beta = (\alpha-1)/(1+\alpha) = 0.15$$

5-11.



THE IDEA IS TO XFORM TO A FRAME WHERE ASTRONAUT IS AT REST BUT ACCELERATING w/ acceleration g IN HIS REST FRAME. USE XFORMS FOR ACCELERATION w/ $u_x' = 0$ $\therefore$

$$a_x = a_x'/\gamma^3$$

$$\gamma^3 a_x = a_x' = g$$

$$\gamma^3 \frac{dv}{dt} = g$$

$$c\gamma^3 \, d\beta = g\,dt$$

$$= g\gamma\, dt' \qquad \text{w/ } t' = \text{Astro TIME}$$

$$\int_0^\beta \frac{c}{g}\frac{d\beta}{1-\beta^2} = \int_0^{T'} dt'$$

$$\frac{c}{2g}\ln\left|\frac{\beta+1}{\beta-1}\right| \Bigg|_0^\beta = T'$$

5-11

$$\frac{c}{2g} \ln\left(\frac{1+\beta}{1-\beta}\right) = T'$$

FOR $\beta = 0.15$,

$$\boxed{\begin{aligned} T' &= 4.63 \times 10^{6} \text{ sec} \\ 1\,\text{yr} &\simeq \pi \times 10^{7} \text{ sec} \\ \Rightarrow T' &= 0.147 \text{ yr} \\ &= 1/7 \text{ yr} \\ T' &\simeq 50 \text{ DAYS OR SO} \end{aligned}}$$

6-9

(a) $E = N h\nu$

$\dot{E} = \dot{N} h\nu$

$= \dot{N} hc/\lambda$

w/ $\dot{N} = 10^{20}$/sec

$h = 6.63 \times 10^{-34}$ J-s

$\lambda = 600$ nm

$c = 3 \times 10^{8}$ m/s

$\boxed{\dot{E} = \text{Power} = 33.2 \text{ W}}$

(b) SEE PHOTON ROCKET DISCUSSION IN TEXT.

E CONS: $m_0c^2 = f\gamma m_0 c^2 + E_r$

p CONS: $0 = f\gamma m_0 v - E_r/c$

$E_r = \dot{E} \cdot 10$ yrs w/ 10 yr $= 10 \times \pi \times 10^{7}$ sec

f = FRACTION OF ORIGINAL LASER MASS THAT "SURVIVES", $f < 1$

6-9

(b) CONT

Now, FROM ABOVE

$$f \gamma m_0 c^2 v = E_r c$$

$$f m_0 c^2 = \frac{E_r c}{\gamma v} = E_r / \beta\gamma$$

$$\Rightarrow \quad m_0 c^2 = E_r/\beta \quad + E_r$$

$$= E_r \left(1 + \frac{1}{\beta}\right)$$

$$m_0 c^2 / E_r = 1 + 1/\beta$$

$$* \quad \boxed{E_r / (m_0 c^2 - E_r) = \beta}$$

WHAT IS E_r?

$$E_r = \dot{N}\, T\, hc/\lambda$$

$$= 10^{20} \times 10\pi \times 10^{7} * 6.63 \times 10^{-34} * 3 \times 10^{8} / 100 \times 10^{-9}$$

$$= 1 \times 10^{10} \text{ J}$$

$$m_0 c^2 = 9 \times 10^{17} \text{ J} \qquad w/m = 10 \text{ kg}$$

6-9

(b) CONT.

$$\Rightarrow \beta \simeq \frac{E_r}{m_0 c^2} = 1.1 \times 10^{-8}$$

$$\Rightarrow \boxed{v = 3.3 \ m/s}$$

(c) DOPPLER EFFECT

$$\nu = \nu_0 \left(\frac{1-\beta}{1+\beta}\right)^{1/2} \qquad \text{SEPARATING}$$

$$\nu = \nu_0 \left(\frac{1+\beta}{1-\beta}\right)^{1/2}$$

$$\dot{E} = \dot{N} \, h\nu$$

$$\frac{\Delta \dot{E}}{\dot{E}_0} = \frac{\dot{N} \, (h\nu - h\nu_0)}{\dot{N} \, h\nu_0}$$

$$= \frac{\nu}{\nu_0} - 1$$

$$= \left(\frac{1-\beta}{1+\beta}\right)^{1/2} - 1$$

$$\simeq 1 - \beta \quad - 1 \qquad (\beta \ll 1)$$

6-9

(c) AND USING $(1-x)^{1/2} \simeq 1-\frac{x}{2}$, $x \ll 1$

$(1+x)^{-1/2} \simeq 1-\frac{x}{2}$, $x \ll 1$

$$\Delta \dot{E}/\dot{E}_0 \simeq -\beta$$

$$\simeq -1.1 \times 10^{-8}$$

$\Rightarrow \Delta \text{Power} \simeq -1.1 \times 10^{-8} * 37.2\ W$

$$\boxed{\Delta P \simeq -0.35\ \mu W}$$

BOOK IS WRONG BY FACTOR OF 2.

6-12.

TRANSFORM TO CM FRAME.
WILL NEED γ_T FOR THIS XFORM.
ALSO, USE THE FACT THAT THE SQUARE OF A 4-VECTOR IS A LORENTZ INVARIANT.

IN CM FRAME BOTH $\bar{p}$ & p HAVE EQUAL & OPPOSITE MOMENTUM.
XFORM TO CM-FRAME CAUSES p TO SPEED UP.

USE ENERGY-MOMENTUM 4-VECTOR.
LINEAR MOMENTUM EQUALS ZERO IN CM-FRAME.

$$4\gamma_T^2 (mc^2)^2 = (E_1 + mc^2)^2 - P_1^2 c^2$$

$$= E_1^2 + 2E_1 mc^2 + (mc^2)^2 - P_1^2 c^2$$

$$4\gamma_T^2 (mc^2)^2 = 2E_1 mc^2 + 2(mc^2)^2$$

$$2\gamma_T^2 = E_1/mc^2 + 1$$

$$\gamma_T = \left[\frac{E_1 + mc^2}{2mc^2}\right]^{1/2}$$

w/ E_1 = TTL ENERGY OF $\bar{p}$ IN LAB FRAME

6-12.

NOW, $E_1 = KE + mc^2$

$= \frac{2}{3}\,GeV + 1\,GeV$

$= 5/3\ GeV$

$\Rightarrow \gamma_T = \left[\frac{5/3 + 1}{2}\right]^{1/2}$

$= 1.155$

$\Rightarrow \beta = \left[1 - 1/\gamma^2\right]^{1/2} = 0.5$

CM: $E_\gamma^{cm} = \gamma_T\, m_p c^2$ (PHOTON GETS ALL ENERGY OF A PROTON)

$P_\gamma^{cm} c = E_\gamma^{cm} = (\gamma_T m_p c)c$

XFORM BACK TO LAB. ALTHOUGH BOTH PHOTONS MOVE w/ VELO = C, THEY WILL HAVE DIFFERENT MOMENTA/ENERGIES

6-12

$$\begin{pmatrix} E \\ P_x c \\ P_y c \\ P_z c \end{pmatrix}' = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ P_x c \\ P_y c \\ P_z c \end{pmatrix}$$

$$\Rightarrow E' = \gamma(E - \beta P_x c)$$

$$= \gamma_T(\gamma_T m_p c^2 - \beta_T \gamma_T m_p c^2)$$

$$= \gamma_T^2 m_p c^2 (1 - \beta_T)$$

$$= 1.155^2 * 1\ \text{GeV}\ (1 - 0.5)$$

$$\boxed{E'_{low} = 667\ \text{MeV} = \tfrac{2}{3}\ \text{GeV}}$$

$$E'_{H} = \gamma_T^2 m_p c^2 (1 + \beta_T)$$

$$= 1.155^2 * 1\ \text{GeV}\ (1 + 0.5)$$

$$\boxed{E'_{H} = 2.00\ \text{GeV}}$$

TEC 1

$E = mc^2$

$E =$ 20 ktons of HE

$= \cancel{20 \times 4.184 \times 10^9}$

$20 \times 10^3 \times 4.184 \times 10^9$ Joules

$m = E/c^2$

$= 9.3 \times 10^{-4}$ kg

$\simeq 10^{-3}$ kg

$m \simeq 1$ gm

= 1/3 MASS OF PENNY