WAVES REVIEW.

NORMAL MODES

$$\left[m^{-1} k - \omega^2 I \right] A = 0$$

FIND ω^2:

$$\det \left[m^{-1} k - \omega^2 I \right] = 0$$

K MATRIX: $k_{ij} = -F_{ij}/x_j$

FORCE ON i BY j

- HOLD i, MOVE j BY x_j.
- DETERMINE FORCE ON i
- DIVIDE BY x_j AND MULTIPLY BY -1.

M IS DIAGONAL W/ MASSES ALONG DIAGONAL. WE DEAL ONLY W/ EQUAL MASSES

INVERSE OF A MATRIX B

$$B^{-1} = \frac{B_c^T}{|B|}$$

$|B|$ ← DETERMINANT

WAVES REVIEW. (NORMAL MODES)

$(B_c)_{ij} = (-1)^{i+j} \, [$DETERMINANT OF MATRIX w/ ROW i AND ROW j DELETED$]$

T MEANS TRANSPOSE (ROW #'s GET SWAPPED w/ COLUMN #'s).

MOTION OF SYSTEM

$$X(t) = \sum_{\alpha=1}^{n} \underbrace{b_\alpha A^\alpha \cos \omega_\alpha t + c_\alpha A^\alpha \sin(\omega_\alpha t)}_{\text{REAL}}$$

$$= \sum_{\alpha=1}^{n} \underbrace{d_\alpha A^\alpha \cos(\omega_\alpha t - \theta_\alpha)}_{\text{REAL}}$$

b_α & c_α (or d_α & θ_α) DETERMINED FROM INITIAL CONDITIONS.

WAVES.

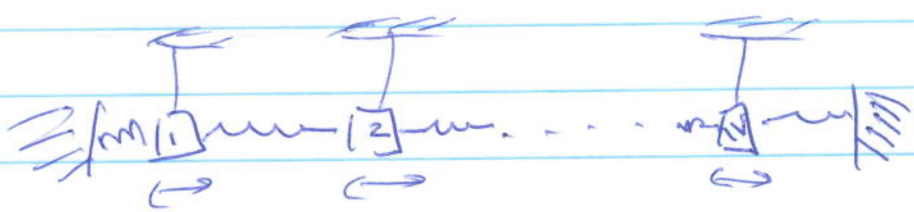
SYMMETRIES.

SOLVE EIGENVALUE PROBLEM

$$S A^{n} = \beta_{n} A^{n}$$

SEE EXAMPLE IN TEXT

SPACE TRANSLATION INVARIANCE.



NORMAL MODES

$$A_j^{n} = \sin \frac{j n \pi}{N+1}, \quad n = 1, 2, \ldots N$$

$$j = 1, 2, \ldots N$$

n LABELS NORMAL MODES

j LABELS BLOCKS.

EIGENFREQUENCIES:

$$\omega^2 = 2B - 2C \cos\left(\frac{n\pi}{N+1}\right)$$

$$\text{w/ } 2B = mg/\ell + 2k \qquad C = K$$

WAVES. REVIEW.

NORMAL MODES FOR INFINITE SYSTEM.

$$A(x) = e^{ikx} \qquad \text{w/} \quad -\frac{\pi}{a} < \mathrm{Re}\,k \le \pi/a$$

$$A(x+\lambda) = A(x) \qquad \text{w/} \quad \lambda = 2\pi/k$$

FINITE $A^n(x) = \sin kx$ w/ $k = n\pi/L$

$L = (N+1)a$

$n = 1, 2, \dots N$

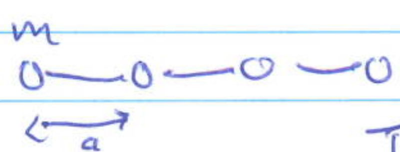
NORMAL MODES

DISPERSION RELATION

EQN THAT RELATES WAVE NUMBER k AND ANGULAR FREQUENCY ω.

EXAMPLE: INFINITE BEADED STRING

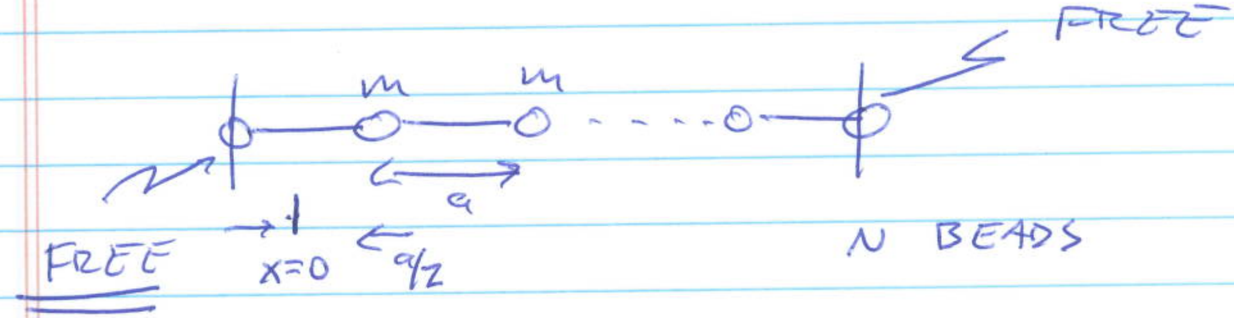
$$\omega^2 = \frac{4T}{ma}\sin^2 ka/2$$



T = STRING TENSION.

WAVE REVIEW

BEADED STRING

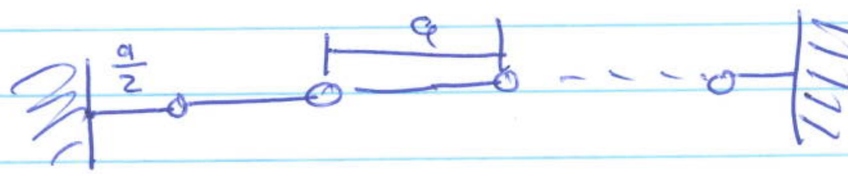


$$A_j = \cos\left[ka\left(j - \tfrac{1}{2}\right)\right], \quad k = \frac{n\pi}{Na}$$

$$\text{w/} \quad \omega^2 = \frac{4T}{ma}\sin^2 ka/2 \qquad \begin{array}{l} n = 0, 1, \ldots N-1 \\ j = 1, \ldots N \end{array}$$

A_j DESCRIBES TRANSVERSE DISPLACEMENT FROM EQUILIBRIUM.

FIXED ENDS



$$A_j^n = \sin\frac{n\pi\left(j - \frac{1}{2}\right)}{Na} = \sin\left[ka\left(j - \tfrac{1}{2}\right)\right]$$

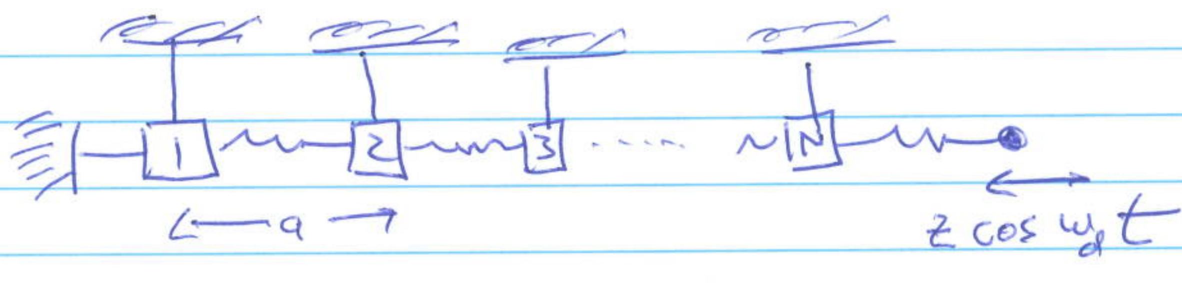
$$\text{w/} \quad k = \frac{n\pi}{Na}$$

$$n = 0, 1, 2, \ldots N-1$$
$$j = 1, 2, \ldots N$$

WAVE REVIEW.

DRIVEN/FORCED OSCILLATORS

FIXED



$$\psi(x,t) = y \sin kx \, e^{-i\omega_d t}$$

$$\text{w/ } y = \frac{z}{\sin kL}$$

$$L = (N+1)a$$

$$k = \frac{1}{a} \cos^{-1} \frac{2B - \omega_d^2}{2C}$$

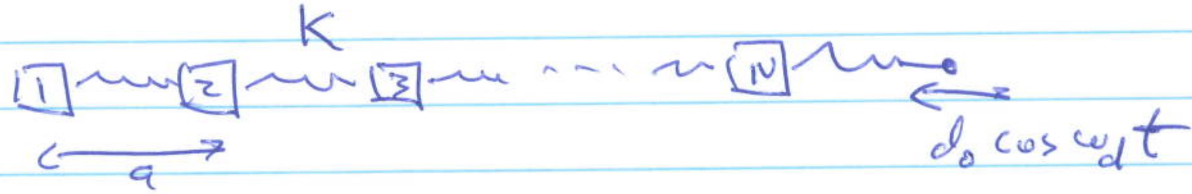
spring

w/ B & C: $2B = mg/\ell + 2k$

$C = k$ (SPRING CONST.)

WAVE REVIEW.

FREE END FORCED OSCILLATION.



$$\psi_s(t) = \frac{\cos\left[\frac{(2s-1)\,ka}{2}\right]}{\cos\left(\frac{2N+1}{2}\right)ka}$$

s = block label

N = # BLKS

$$\omega_d^2 = \frac{4K}{m}\sin^2 ka/2$$

WAVE REVIEW

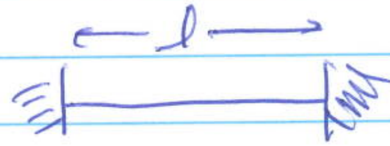
CONTINUOUS STRING

WAVE EQN:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{T}{\rho_L} \frac{\partial^2 y(x,t)}{\partial x^2}$$

w/ DISPERSION RELATION $\omega^2 = \frac{T}{\rho_L} k^2$

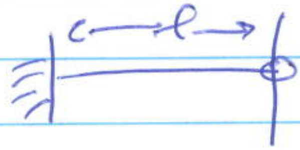
FIXED ENDS



$$y(x) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{\ell}$$

$$c_n = \frac{2}{\ell} \int_0^{\ell} dx \sin \frac{n\pi x}{\ell} \, y(x)$$

1 FREE END



$$y(x) = \sum_{n=1}^{\infty} c_n \sin \frac{(2n-1)\pi x}{2\ell}$$

$$c_n = \frac{2}{\ell} \int_0^{\ell} dx \sin \frac{(2n-1)\pi x}{2\ell} \, y(x)$$