WAVES

REVIEW

\( n = \text{Longitudinal modes in massive spring} \)

\[
\omega^2 = \frac{K l}{l} k^2
\]

\( k = \text{Spring constant of entire spring} \)

\( l = \text{Spring length} \)

\( \rho = \text{Linear mass density of spring} \)

\( \text{Fixed ends} \rightarrow \begin{array}{c}
\end{array} \)

\[
A_n(x) = \sin \frac{n \pi x}{l}
\]

\[
\omega_n = \sqrt{\frac{k - \rho}{\rho}} k_n
\]

\( n = 1, 2, \ldots, \infty \)

\( A_n(x) \) \text{ describes displacement of spring from its equilibrium position } x. \)
WAVES REVIEW

FREE END

\[ A_n(x) = \sin \left( \frac{(2m-1)\pi x}{2l} \right), \quad m = 1, 2, \ldots \]

\[ \kappa_m = \frac{(2m-1)\pi}{2l} \]

SAME DISPERSION RELATION AS FOR FIXED - FIXED ENDS

MASS ON LIGHT SPRING

\[ \varepsilon = \frac{F}{m} \]

\[ \varepsilon \ll 1 \Rightarrow "\text{LIGHT SPRING}" \]

\[ y(x, t) = A \sin \kappa_m x \cos \omega t \]

DISPLACEMENT FROM EQUILIBRIUM

POSITION X. A IS SOME AMPLITUDE DEPENDS ON INITIAL CONDITIONS.
WAVES REVIEW

DISPERSION RELATION

\[ \omega_n = n \pi \sqrt{\frac{k}{t}} \quad n = 1, 2, \ldots \infty \]

\[ \omega_0 = \sqrt{\frac{k}{\mu}} \quad (\omega_n \gg \omega_0) \]

\[ k_m = \frac{m \pi}{\ell} \]

\[ k_0 = \frac{\nu}{\ell} \]

\[ \varepsilon \]

\[ \nu(x,t) = A \sin \nu \pi x \cos \omega_0 t \]

\[ \lambda \nu(x,t) = A \sin \frac{n \pi x}{\ell} \cos \omega_0 t, \quad n = 1, 2, \ldots \]
WAVES REVIEW

TRAVELING WAVES & STANDING WAVES

TRAVELING WAVE TO RIGHT: \( y(x,t) = \cos (kx - wt) \)

TO LEFT: \( y(x,t) = \cos (kx + wt) \)

STANDING WAVE = SUPERPOSITION OF 2 WAVES
ONE TRAVELING TO LEFT
OTHER TO RIGHT

\[ \cos kx \cos \omega t = \frac{1}{2} \left[ \cos (kx - \omega t) + \cos (kx + \omega t) \right] \]

STANDING WAVE

FORCE, POWER IMPEDANCE

FORCE ON STRING \( F = -T \frac{\partial y(x,t)}{\partial x} \)

\( T = \text{STRING TENSION} \)
Power needed to produce a traveling wave in a string with tension $T$:

$$P = \frac{1}{2} A^2 \omega^2 \sin^2 \omega t$$

- Amplitude $A$
- Linear mass density $\rho$
- $T = \rho L$

$$\langle P \rangle = \frac{1}{2} A^2 \omega^2 \frac{1}{2}$$