WAVES

REVIEW.

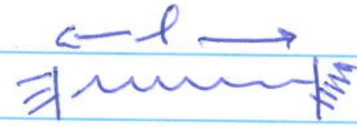
(c) LONGITUDINAL MODES IN MASSIVE SPRING

DISPERSION RELATION: $\omega^2 = \dfrac{K\ell}{\rho_L} k^2$

K = SPRING CONSTANT OF ENTIRE SPRING
ℓ = SPRING LENGTH

ρ_L = LINEAR MASS DENSITY OF SPRING

<u>FIXED ENDS.</u>



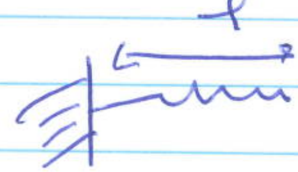
$$A_n(x) = \sin \frac{n\pi x}{\ell} \qquad w/\ k_n = n\pi/\ell$$

$$\omega_n = \sqrt{\frac{K\ell}{\rho_L}}\, k_n \qquad n = 1, 2, \ldots \infty$$

$A_n(x)$ DESCRIBES DISPLACEMENT OF SPRING FROM ITS EQUILIBRIUM POSITION X.

WAVES REVIEW

FREE END



$$A_n(x) = \sin \frac{(2n-1)\pi x}{2\ell}, \quad n = 1, 2, \ldots \infty$$

$$k_n = \frac{(2n-1)\pi}{2\ell}$$

SAME DISPERSION RELATION AS FOR FIXED-FIXED ENDS

MASS ON LIGHT SPRING

$$\varepsilon \equiv \frac{\rho_\ell \ell}{m}$$

$\varepsilon \ll 1 \Rightarrow$ "LIGHT SPRING

$$\psi(x,t) = A \sin k_n x \cos \omega_n t$$

DISPLACEMENT FROM EQUILIBRIUM POSITION X. A IS SOME AMPLITUDE. DEPENDS ON INITIAL CONDITIONS.

WAVES REVIEW.

DISPERSION RELATION.

$$\omega_n \simeq n\pi \sqrt{\frac{K}{\rho_L \ell}} \quad , n = 1, 2, \ldots \infty$$

$$\omega_0 \cong \sqrt{K/m} \qquad (\omega_n \gg \omega_0)$$

$$k_n \simeq n\pi/\ell$$

$$k_0 \simeq \frac{\sqrt{\varepsilon}}{\ell}$$

so,

$$\psi(x,t) \cong A \sin\sqrt{\varepsilon} \cos\omega_0 t$$

&

$$\psi(x,t) \cong A \sin\frac{n\pi x}{\ell} \cos\omega_n t \quad , n = 1, 2 \ldots \infty$$

WAVES REVIEW

TRAVELING WAVES & STANDING WAVES

TRAVELING WAVE TO RIGHT: $\psi(x,t) \propto \cos(kx - \omega t)$

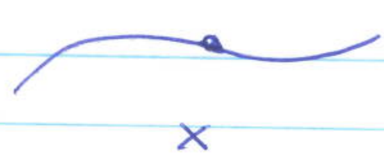
TO LEFT: $\psi(x,t) \propto \cos(kx + \omega t)$

STANDING WAVE = SUPERPOSITION OF 2 WAVES, ONE TRAVELING TO LEFT, OTHER TO RIGHT

$$\underbrace{\cos kx \cos \omega t}_{\text{STANDING WAVE}} = \frac{1}{2}\left[\cos(kx - \omega t) + \cos(kx + \omega t)\right]$$

FORCE, POWER IMPEDANCE

FORCE ON STRING $F = -T \dfrac{\partial \psi(x,t)}{\partial x}$



T = STRING TENSION

WAVES REVIEW

POWER REQ'D TO PRODUCE A TRAVELING WAVE IN A STRING W/ TENSION T:

$$P = Z A^2 \omega^2 \sin^2 \omega t$$

AMPLITUDE

$$Z = \sqrt{\rho T}$$

linear MASS DENSITY.

$$= T k/\omega$$

$$\langle P \rangle = Z A^2 \omega^2 / 2$$