Homework 5

1. We examined damped, driven harmonic oscillators in lecture. Since such systems have damping, energy must be expended by the “driver” to keep whatever is “driven,” oscillating. Let’s try to calculate the time rate of energy (or power) expended by the driver. This quantity is also the power absorbed by the driven system. Problem 4-10 in French explores this topic. Notice that the “b” in French’s notation corresponds to “2βm” in ours. Also, for the $Q$ of the oscillatory system, you can use the result $Q = \omega_0/2\beta$ (our notation). These are details, but details matter. Also, recall that the long term solution for a damped, driven harmonic oscillator is $x(t) = A(\omega)\cos(\omega t - \delta)$, where $\omega$ is the angular frequency of the driver and the amplitude $A$ of the oscillation is a function of $\omega$. Take it from there.

2(a). For a driven oscillator of mass $m$, show that the energy dissipated per cycle $E'$ by a frictional force $F_d = -bv$ at frequency $\omega$ and amplitude $A$ equals $E' = \pi b \omega A^2$. Be sure to check your units.

2(b). What is the ratio of the energy dissipated per cycle to the stored energy? This should be a simple ratio involving $\omega$, $b$, $m$ and maybe some numbers.

2(c). Show that at the resonance frequency of a lightly damped oscillator,

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\frac{\text{energy dissipated/cycle}}{\text{stored energy}} = \frac{2\pi}{Q}.
\]

3. For a driven, damped oscillator, plot $A(\omega)$, where $\omega$ is the driving angular frequency. Do this by plotting $\omega$ as a multiple of $\omega_0$, the natural frequency of the oscillator. You will need to do this for several values of $Q$. (How about $Q = 2.5$, 5 and 10)? I flashed the plot in lecture (and a similar plot is in French on p. 99). Now is your chance to plot it yourself. Any plotting routine is OK. A xeroxed or scanned version of somebody else’s is not.