PHYS 3310
Spring 2010
TE Coan
Due: 31 Mar '31, 6:00 pm

Homework 5

1. We examined damped, driven harmonic oscillators in lecture. Since such systems have damping, energy must be expended by the "driver" to keep whatever is "driven," oscillating. Let's try to calculate the *time rate* of energy (or power) expended by the driver. This quantity is also the power *absorbed* by the driven system. Problem 4-10 in French explores this topic. Notice that the "b" in French's notation corresponds to "$2\beta m$" in ours. Also, for the Q of the oscillatory system, you can use the result $Q = \omega_0/2\beta$ (our notation). These are details, but details matter. Also, recall that the long term solution for a damped, driven harmonic oscillator is $x(t) = A(\omega)\cos(\omega t - \delta)$, where ω is the angular frequency of the driver and the amplitude A of the oscillation is a function of ω. Take it from there.

2(a). For a driven oscillator of mass m, show that the energy dissipated per cycle E' by a frictional force $F_d = -bv$ at frequency ω and amplitude A equals $E' = \pi b \omega A^2$. Be sure to check your units.

2(b). What is the ratio of the energy dissipated per cycle to the stored energy? This should be a simple ratio involving ω, b, m and maybe some numbers.

2(c). Show that at the resonance frequency of a lightly damped oscillator,

$$\frac{\text{energy dissipated/cycle}}{\text{stored energy}} = \frac{2\pi}{Q}$$

.

3. For a driven, damped oscillator, plot $A(\omega)$, where ω is the driving angular frequency. Do this by plotting ω as a multiple of ω_0, the natural frequency of the oscillator. You will need to do this for several values of Q. (How about $Q = 2.5$, 5 and 10)? I flashed the plot in lecture (and a similar plot is in French on p. 99). Now is your chance to plot it yourself. Any plotting routine is OK. A xeroxed or scanned version of somebody else's is not.