PHYS 3310
Spring 2010
TE Coan
Due: 8 Apr '31, 6:00 pm

Homework 6

1. Consider the simple system of coupled oscillators comprised of 5 identical beads each of mass m attached to a flexible string, with each bead separated from its nearest neighbor by a distance l. The string is oriented along a horizontal line and the beads are allowed to oscillate up and down with small amplitude (i.e., consider transverse oscillations only). The beads at each far end are fixed so they do not move even a smidgin'.

(a) How many normal modes M are there?

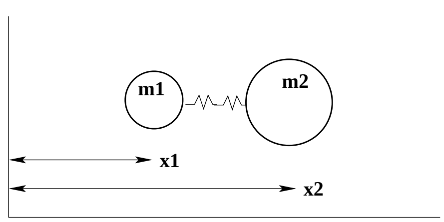
(b) Channel your inner Picasso and sketch the normal modes with the 3 lowest frequencies. Make sure you identify which normal mode goes with the lowest normal frequency, which with the next highest frequency, etc. Use graph paper so that your plot is sensible. Print your own graph paper if you don't own any or want to purchase any. Don't use notebook paper since the sparse line spacing will make the sketch look lousy.

(c) Calculate the frequencies corresponding to the normal modes with the 3 lowest frequencies (i.e., ω_1, ω_2 and ω_3). Express your answers as something times the frequency $\omega_0 = \sqrt{T/lm}$, where T is the string tension.

2(a). French 5-9. For part (b) of 5-9, the observed ratio of frequencies for the two modes is 2349/1288. Compare your answer.

2(b). Speaking of CO_2, approximately what mass fraction f_{CO_2} of the Earth's atmosphere is composed of this molecule, without which there would be no life on the planet? (This has nothing per se to do with oscillatory motion but does have something to do with basic science literacy and knowledge of the world around you.)

3. Let us see if we can understand something about how a $NaCl$ molecule works. Model it as two masses attached by a spring and we'll pay attention to its *vibrational* mode of oscillation and ignore any rotational motion. The diagram below sets the stage.



Let the masses of the twp atoms be m_1 and m_2, and label the effective spring constant as s. Call x the extension of the spring beyond its relaxed length l. If x_1 and x_2 are the positions of the center-of-mass of each of the two atoms, then the equations of motion of the two masses are:

$$m_1 \ddot{x}_1 = sx$$

$$m_2 \ddot{x}_2 = -sx.$$

(a) Combine these guys into a single equation to show that the system oscillates with a frequency $\omega_0^2 = s/\mu$, where the reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$.

(b) The observed vibrational frequency f of $NaCl$ is $f = 1.14 \times 10^{13}$ Hz. What then is the effective interatomic spring constant s? Use units of N/m. You will also need that the atomic mass of Na $= 23$ a.m.u and that the atomic mass of Cl $= 35$ a.m.u. Recall that 1 a.m.u. $= 1.67 \times 10^{-27}$ kg. Be careful about factors of 2π and the distinction between frequency and *angular* frequency..