

Homework 7

1. Consider two identical point sources of a monochromatic spherical electromagnetic wave (e.g., two antennas). By identical, I mean they have the same source strength and emit spherical electromagnetic waves of the same angular frequency ω . They are separated from each other by a distance d and the antennas are parallel. For those of you who attend class, this setup is familiar.

(a) In the mid-plane of the antennas, perpendicular to the antennas, and at some arbitrary distance R away from the midpoint between the antennas, what is the expression for the angular pattern of the intensity I as a function of phase difference δ between the two sources? Let I_0 be the intensity from one source alone at the arbitrary distance R . Define all quantities used so the reader knows what you are talking about. Who knows, maybe a figure might be useful.

(b) What value of d and what value of phase offset difference $\Delta = \phi_1 - \phi_2$ produces the figure B&B 8.4a (p529). Be careful. We use a slightly different sign convention than B&B! Convince me you just didn't copy from a book by making a plot of the curve. Do not xerox the curve or draw it by hand. Note: You will need to make a polar plot. In gnuplot, you will need to issue the commands `set polar`, `set radians` and `set samples 1000, 1000`. (This latter just makes the plot smoother by sampling at more points.) In other plotting routines, you will need to do something similar.

2. A *free electron laser* is a device that can produce a very short light pulse. If the width Δt of the light pulse is $\Delta t = 100$ fs ($= 100 \times 10^{-15}$ s) and the central wavelength is 500 nm, estimate the spread of wavelengths $\Delta\lambda$ in the light pulse. Remember your units or kiss your points goodbye.

3. Two infrared lasers have nearly the same wavelength and produce a beat frequency if they are incident on a photodetector with a sufficiently fast response time. One of the lasers has a wavelength $\lambda_1 = 766.49110$ nm while a second laser has a slightly shorter wavelength. They produce a beat frequency $f_b = 462$ MHz. What is the wavelength λ_2 of the second laser? No shoes, no service. No units, no credit.

4. Surf's up, baby. Surface water waves are subject to two types of forces that tend to flatten the water's surface: gravity and surface tension. The relative strength of these forces depends upon the wavelength of the waves. For waves in deep water (deep means the depth of the water greatly exceeds the wavelength of the water wave), the relation between the angular frequency and wave number of the wave are related by the dispersion

relation

$$\omega^2 = gk + \frac{Sk^3}{\rho},$$

g is the acceleration due to gravity and S and ρ are the surface tension and density of water, respectively. ($S = 7.2 \times 10^{-2}$ N/m and $\rho =$ something you ought to know.)

- a. Deduce the ratio R_s of the group velocity to the phase velocity in the limit of short wavelength.
- b. Deduce the ratio R_l of the group velocity to the phase velocity in the limit of long wavelength.
- c. At what wavelength(s) λ are the phase velocity and group velocity equal?