PHYS 3340
Spring 2009
TE Coan
Due: 6 Mar '09, 6pm via email.

Homework 5

1. Use `orbit.cc` to verify Kepler's 3rd law, $\tau^2 \propto a^3$, where τ is a planet's period and a is the semi-major axis of its elliptical orbit. Do this by machine computing τ and a. Make a short table listing 5 values of τ v. a.Use AU for the distance unit and years for the time unit. Do **not** use τ and a values for Earth. (I know what they are.) Also, have some reasonable spread in your choices of a. You will need to use $GM = 4\pi^2\,\mathrm{AU}^3/\mathrm{yr}^2$. Make sure your orbits close.

2. Modify `orbit.cc` for the case where the gravitational force f between M and m is given by $f = -kGMm/r^3$. For simplicity, set $kGM = 1$ in your code. For initial conditions that produced one of the ellipses in question 1, what is the shape of the orbit for m? Make sure your time steps are long enough so that you get a sense of the curve. Include code and a plot in your answer file.

3. Compute the period T of oscillation of a realistic pendulum. Assume that you have a bob of mass m suspended by a thin string of length l measured between support point and the bob's center of mass. From PHYS 1303 we know that for "small" displacements from equilibrium, $T = 2\pi\sqrt{l/g}$. For $l = 1\,\mathrm{m}$ this gives $T = 2.01\,\mathrm{sec}$. Suppose you start the pendulum from rest with an initial displacement angle $\theta_0 = 45°$. What is the period (call it $T_{\pi/4}$)? What is T ($T_{\pi/2}$) for $\theta_0 = 90°$? To get some perspective, also write these answers in the form $T_\theta = kT_0$, where $T_0 = 2\pi\sqrt{l/g}$, $\theta = \pi/4, \pi/2$ and k is a number you need to determine from your calculations.

4. The rabbits and the foxes are at it again on some island. The rabbits just want to eat clover (of which the island has a seemingly inexhaustible supply) and the foxes just want to eat the rabbits. We are interested in the population x of the rabbits and of the foxes y as a function of time t. If there were no foxes ($y = 0$), then the population of the rabbits would be governed by $dx/dt = ax$ ($a > 0$), due to the unlimited supply of clover. We can model the interaction of rabbit and foxes per unit time as proportional to xy and then assume that some fraction of the encounters leads to an eaten rabbit. We then have for the time evolution of the rabbit population $dx/dt = ax - bxy$ ($b > 0$).

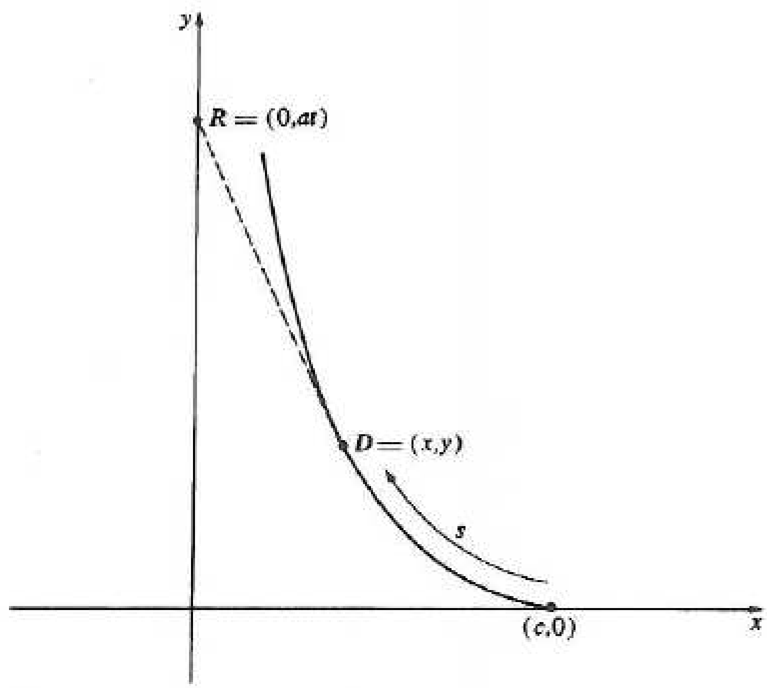
The story with the foxes is similar but not identical. With no rabbits ($x = 0$) to eat, the foxes die off, $dy/dt = -cy$ ($c > 0$). On the other hand, if there are rabbits around to eat, the fox population can increase. The net result is that for the foxes, we have $dy/dt = -cy + dxy$ ($d > 0$), since the growth in the number of foxes depends on encounters with rabbits.

So, we have the so-called *Volterra prey-predator equations*:

$$\begin{cases} dx/dt = & x(a-by) \\ dy/dt = & -y(c-dx) \end{cases}$$

Plot $x(t)$ and $y(t)$ and try to put them on the same plot. Choose some values for a, b, c and d. (I don't know, you might try $a = 1.0, b = 0.5, c = 0.95, d = 0.25$.) You also have to pick some initial values for the rabbit and fox populations. I'll leave this to you. Include source, initial values and a plot in your answer file.

5. A dog is chasing a random car. What it does when it catches the car is anyone's guess. The car starts at the origin and travels along the y-axis with speed a. Simultaneously, the dog starts running with speed b from the point $(c, 0)$ toward the car. What is the path of Rover?



After some time t, the car is at position $R = (0, at)$ and the dog is at position $D = (x, y)$. See the figure. Since the line DR is tangent to the dog's path, we have

$$\frac{dy}{dx} = \frac{y - at}{x}$$

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Differentiating this with to respect to x yields

$$x\frac{d^2y}{dx^2} = -a\frac{dt}{dx}$$

Now, $\frac{dx}{dt}$ is just the x-component of the dog's speed. For a small piece of the dog's path $ds = \sqrt{dx^2 + dy^2}$, so we have

$$\frac{dx}{dt} = -b\cos\theta = -b\frac{dx}{\sqrt{dx^2 + dy^2}} = -\frac{b}{\sqrt{1 + (\frac{dy}{dx})^2}}$$

Finally, we have, with $k = \frac{a}{b}$,

$$x\frac{d^2y}{dx^2} = k\sqrt{1 + (\frac{dy}{dx})^2}.$$

Solve this equation to determine $y = y(x)$, the dog's trajectory. Pick k sensibly. Include source code and plot in answer file.