

## Homework 6

1. Machine find a root to the following functions  $f(x)$  with the associated initial guesses. Also, create a plot that shows  $x_i$  versus iteration  $i$ .

$$\begin{aligned} f(x) &= \tanh(x) & x_0 &= 2.0 \\ f(x) &= \ln(x) & x_0 &= 3.0 \\ f(x) &= \sqrt{(x^2 + 2x + 1)} - 2x \sin(x) & x_0 &= -5.0 \end{aligned}$$

Indicate what the tolerance is for your roots and attach the plots to your answer file.

2. The Newton-Raphson (NR) technique can take a long time or even diverge for functions that have a complicated analytic behavior near a root. For example, compare the number of iterations required by the NR technique to find the root near  $x = 0$  with an accuracy of  $10^{-4}$  for the functions  $\sin(x)$  and  $x\sqrt{|x|}$  when starting from  $x = 0.1$ . Perform the same comparison using the bisection technique. Include relevant data in your answer file.

3. Find the energy eigenvalue associated with bound states for the potential well discussed in lecture. This means you need to solve the equation  $f(E) = \xi \tan \xi - \eta = 0$  for the even parity solutions and  $f(E) = \xi \cot \xi + \eta = 0$  for the odd parity solutions. Here,  $\xi = \alpha a$ ,  $\eta = \beta a$ , as well as  $\beta = \sqrt{-2mE/\hbar^2} = \sqrt{-0.0483E}$  and  $\alpha = \sqrt{2m(E + V_0)/\hbar^2} = \sqrt{0.0483(E + V_0)}$ . As a reminder, energies  $E$  are measured in MeV and distances  $x$  are measured in fm. (Recall that  $V_0 = 83$  MeV and  $a = 2$  fm.) Include code and eigenvalues in your answer file. **Hint:** use gnuplot to sketch the relevant functions so you don't end up hunting vainly for a root bracket. The numerical typo on the lecture 16 slides 3 and 4 has been fixed.