PHYS 3340
Spring 2009
TE Coan
Due: 28 Apr '09, 6pm via email.

Homework 8

1. Sample the odd signal $k(t) = \sin(2\omega t) + 2\sin(3\omega t) + 5\sin(4\omega t)$, with $\omega = 2\pi f$, $f = 10\,\text{Hz}$. What are the ratios of the corresponding H_n for relevant values of n? (Write the ratios in a sensible way.) Using the clunky method of computing the DFT, are the relevant H_n real or imaginary? Include your answers in the answer file along with a plot of the H_n.

2. Sample the function $g(t) = \sin(2\pi f t)$ with $f = 10.5\,\text{Hz}$ at two different sampling frequencies, f_1 and f_2. Use any number of total measurements you want but sample in two ways: f_1 should be *below* the Nyquist critical frequency f_c and f_2 should be *above*.

Compute the DFT of $g(t)$ (using the clunky DFT routine) for both f_1 and f_2. Recreate $g(t)$ from the two DFTs corresponding to f_1 and f_2. How well did you do? Now you see why paying attention to the Nyquist critical frequency is important. Include in your answer file: your values of f_1 and f_2; the plots of the two DFTs corresponding to f_1 and f_2 (indicate which plot goes with which sampling frequency); the plots for $g(t)$ recreated from the DFTs.

3. Gauss determined the orbit of the asteroid Pallas using the observational data:

θ	0	30	60	90
x	408	89	-66	10
θ	120	150	180	210
x	338	807	1238	1511
θ	240	270	300	330
x	1583	1462	1183	804

where θ is the ascension in degrees and x is the declination in minutes. (Ascension and declination are orthogonal coordinates.).

(a) Fit the above data to the function

$$f(\theta) = a_0 + \sum_{k=1}^{5}[a_k \cos(2\pi k\theta/360) + b_k \sin(2\pi k\theta/360)] + a_6 \cos(2\pi 6\theta/360),$$

where $a_k, k = 0, \ldots, 6$, and $b_k, k = 1, \ldots 5$ are parameters to be determined by the fit. (Using the clunky DFT algorithm instead of the slick FFT algorithm will make your life easier since reading the DFT file is more straightforward with the former algorithm.)

Plot the raw data and the fit function on the same graph and include them along with the parameters and source code in your tar file.