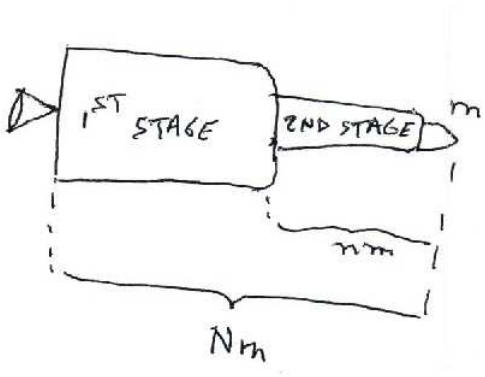
PHYS 3344
Fall 2017
TE Coan
Due: 23 Sep '17 6:00 pm

Homework 4

1. This is a somewhat involved problem, but fully within your ability to solve. It is designed to show you the advantages of a multi-stage rocket compared to a single-stage rocket. Since the commercialization of space by Mr. Amazon and Mr. Tesla seems approaching, it seems sensible to know something about it. Suppose that the payload (e.g., a capsule carrying some hyper-affluent people, or maybe something more interesting) has a mass m and is mounted on a two-stage rocket (see the fig.). The *total* mass –both rocket stages fully fueled, plus the payload – is Nm. The mass of the second-stage plus the payload, after first-stage burnout and separation, is nm. In each case, the ratio of the burnout mass (casing) to initial mass (casing plus fuel) is r, and the exhaust speed of the gas with respect to either stage is v_{ex}.



a) Show that the velocity v_1 gained after the first stage burn, starting from rest (and ignoring gravity), is given by

$$v_1 = v_{ex} \ln \left[\frac{N}{rN + n(1-r)} \right]$$

b) Obtain a corresponding expression for the additional velocity v_2 gained from the second-stage burn. Box your answer.

c) Adding v_1 and v_2, you have the payload velocity v in terms of N, n, and r. Taking N and r as constants, find the value of n that maximizes v. **HINT:** This will turn out to be a simple function of N. The algebra can seem messy so I recommend NOT combining the natural logs when you go to maximize v. You can also use Mathematica. Box your answer.

d) Show that the condition for v to be a maximum corresponds to having equal velocity gains for each of the two stages. Find the maximum value of v. Comment on why this expression makes sense. Box your answer.

e) Find an expression for the payload velocity of a single-stage rocket with the same values of N, r and V_{ex}. Box your answer.