PHYS 3344
Fall 2017
TE Coan
Due: 20 Oct '17 6:00 pm

Homework 6

1. Our text makes the (correct) statement that a force $\mathbf{F}(\mathbf{r})$ that is central and spherically symmetric is conservative. Show that, in sphercial polar coordinates, such a force has the property $\nabla \times \mathbf{F} = 0$.

2. See if you can prove the so-called **virial theorem**. This is a statement that relates the average kinetic energy of a stable system to the potential energy of the system. It applies when the force between two particles of the system has a corresponding potentual energy U of the form $U = kr^n$, where r is the separation of the particles and n is some real number. So, suppose a mass m moves in a circular orbit about the origin in the field of an attractive central force with potential energy $U = kr^n$. Show that the kinetic energy T of the particle is $T = nU/2$. You did something like this in PHYS 1303 but the virial theorem was never mentioned.

3. Suppose the potential energy (PE) U of a mass m a distance r from the origin in an effective one-dimensional universe is given by

$$U(r) = U_0 \left(\frac{r}{R} + \lambda^2 \frac{R}{r} \right),$$

for $0 < r < \infty$, and with U_0, R and λ all positive constants.

a) Find the equilibrium position r_0. Box the answer.

b) If you call x the distance from the equilibrium position, show that for small x, the PE has the form $U = \text{const} + \frac{1}{2}kx^2$. What is the angular frequency ω_0 of small oscillations? Box your answers.

4. In class we never talked too much about oscillators in more than one dimension (unless they were radial oscillators.) However, what we learned carries over to such oscillators. So-called **isotropic** oscillators have the same spring constant in each of the dimensions they oscillate in. So-called **anisotropic** oscillators can have different spring constants, and hence oscillation frequencies, for each of their dimensions. Consider a two-dimensional anisotropic oscillator. Its motion is described by Eq. (5.23) in Taylor.

a) Show that if the ratio of frequencies is rational (i.e., $\omega_x/\omega_y = p/q$ where p and q are integers) then the motion is periodic. What is the period T? Box that answer.

b) Show that if the ratio of frequencies is irrational, then the motion never repeats itself.