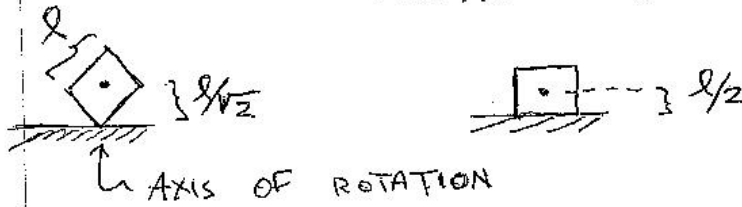


M. 11.11

FRICTION CASE



$E = T + U = \text{CONST}$ (GRAVITY IS ONLY NET EXTF)

$$\text{AT } t = 0 : E = U = mg \frac{l}{\sqrt{2}}$$

$$\text{AT } t = t_f : E = \frac{1}{2} I \omega^2 + mg \frac{l}{2}$$

FROM EXAMPLE 11.5 AND STEINER'S PARALLEL AXIS THM

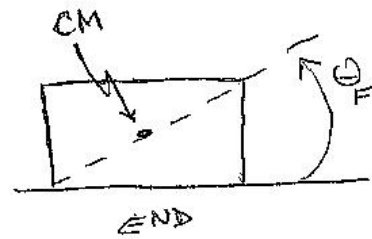
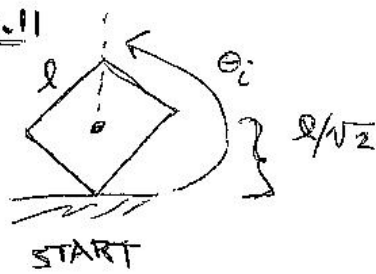
$$\begin{aligned} I &= \frac{1}{6} M l^2 + M \frac{l^2}{2} \\ &= \frac{2}{3} M l^2 \end{aligned}$$

$$\text{SO, } mg \frac{l}{\sqrt{2}} = \frac{1}{3} M l^2 \omega^2 + mg \frac{l}{2}$$

$$g \left(\frac{1}{\sqrt{2}} - \frac{1}{2} \right) = \frac{1}{3} l \omega^2$$

$$\Rightarrow \omega^2 = \frac{3g}{l} \left(\frac{1}{\sqrt{2}} - \frac{1}{2} \right)$$

M11.11



$$y_{cm} = \frac{l}{\sqrt{2}} \sin \theta$$

$$\dot{y}_{cm} = \frac{l \dot{\theta}}{\sqrt{2}} \cos \theta$$

$$\dot{y}_{cm} \Big|_{\theta=45^\circ} = \frac{l \dot{\theta}}{2}$$

Cons. of E:

$$\begin{aligned} mg \frac{l}{\sqrt{2}} &= mg \frac{l}{2} + \frac{1}{2} M \dot{y}_{cm}^2 + \frac{1}{2} I \omega^2 \\ &= mg \frac{l}{2} + \frac{1}{2} m \frac{l^2 \omega^2}{4} + \frac{1}{2} \left(\frac{1}{6} M l^2 \right) \omega^2 \end{aligned}$$

$$\begin{aligned} g l \left(\frac{1}{\sqrt{2}} - \frac{1}{2} \right) &= \frac{1}{8} l^2 \omega^2 + \frac{1}{12} l^2 \omega^2 \\ &= \frac{5}{24} l^2 \omega^2 \end{aligned}$$

$$\boxed{\omega^2 = \frac{24}{5} \frac{g}{l} \left(\frac{1}{\sqrt{2}} - \frac{1}{2} \right)}$$

NOTE THAT $\omega_{SLIDE} > \omega_{FRICTION}$

M 11.12

SEE EQ (11.13b) IN TEXT.

$$I_{11} = \sum_d m_d (r_d^2 - x_d^2)$$

$$I_{22} = \sum_d m_d (r_d^2 - y_d^2)$$

$$I_{33} = \sum_d m_d (r_d^2 - z_d^2)$$

$$\begin{aligned} I_{11} + I_{22} &= \sum_d m_d \left[(r_d^2 - x_d^2) + (r_d^2 - y_d^2) \right] \\ &= \sum_d m_d (y_d^2 + z_d^2 + x_d^2 + z_d^2) \\ &= \sum_d m_d (r_d^2 + z_d^2) \\ &= \sum_d m_d \left[(r_d^2 - z_d^2) + 2z_d^2 \right] \\ &= I_{33} + 2 \sum_d m_d z_d^2 \\ &\geq I_{33} \end{aligned}$$

$$\text{HENCE, } I_{33} \leq I_{11} + I_{22}$$

CYCLICALLY PERMUTE OTHER INDICES

$$I_{11} \leq I_{22} + I_{33}$$

$$I_{22} \leq I_{33} + I_{11}$$

\therefore NO PRINCIPAL MOMENT EXCEEDS SUM OF OTHER TWO.