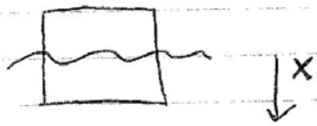
PHYS 3344 HW SOLUTIONS

M3.7



SUPPOSE OBJECT IS FIRST IN EQUILIBRIUM AND IS THEN DISPLACED DOWNWARD BY AN AMOUNT X, THE RESTORING FORCE IS THE BUOYANT FORCE DUE TO THE VOLUME OF FLUID DISPLACED.

$$m\ddot{x} = F_B = \rho_0 V g \qquad \text{BUOYANT FORCE UPWARDS}$$
$$= -\rho_0 A g x$$

BUT $m = \rho V_{OBJ}$, SO

$$\rho V_{OBJ} \ddot{x} = -\rho_0 A g x$$

$$\ddot{x} + \rho_0 A g/(\rho V_{OBJ})\, x = 0 \qquad \text{SHM EQN.}$$

AT EQUILIBRIUM $(x=0)$:

$$mg = \rho_0 V g$$

$$\Rightarrow \rho V_{OBJ} = \rho_0 V$$

REWRITE SHM EQN

M 3.7 CONT.

$$\ddot{x} + \rho_0 A g/(\rho_0 V)\, x = 0$$

$$\ddot{x} + (Ag/V)\, x = 0$$

$$\Rightarrow \omega^2 = Ag/V$$

OR

$$T = 2\pi/\omega$$

$$\boxed{T = 2\pi \left[\frac{V}{gA}\right]^{1/2}}$$

$$\boxed{T \simeq 0.18 \text{ SEC}}$$

M 3.13

NEED TO SOLVE $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$

FOR CASE WHEN $\beta^2 = \omega_0^2$.

TRY $x(t) = y(t)\exp(-\beta t)$.

$$\dot{x} = \dot{y}e^{-\beta t} - \beta y e^{-\beta t}$$

$$\ddot{x} = \ddot{y}e^{-\beta t} - \beta\dot{y}e^{-\beta t} - \beta\dot{y}e^{-\beta t} + \beta^2 y e^{-\beta t}$$

$$= \ddot{y}e^{-\beta t} - 2\beta\dot{y}e^{-\beta t} + \beta^2 y e^{-\beta t}$$

NOW,

$$\ddot{x} + 2\beta\dot{x} = \ddot{y}e^{-\beta t} - 2\beta\dot{y}e^{-\beta t} + \beta^2 y e^{-\beta t}$$

$$+ 2\beta\dot{y}e^{-\beta t} - 2\beta^2 y e^{-\beta t}$$

$$\omega_0^2 x = \omega_0^2 y e^{-\beta t}$$

$$\Rightarrow \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = \ddot{y}e^{-\beta t} - \beta^2 y e^{-\beta t} + \omega_0^2 y e^{-\beta t} = 0$$

$$\left[\ddot{y} - \beta^2 y + \omega_0^2 y\right]e^{-\beta t} = 0$$

$$\left[\ddot{y} + (\omega_0^2 - \beta^2)y\right]e^{-\beta t} = 0$$

$\Rightarrow \ddot{y} = 0$ (FOR $\omega_0^2 = \beta^2$), SINCE $e^{-\beta t} \neq 0$

INTEGRATE, $y = A + Bt$

SO, $\boxed{x(t) = (A + Bt)e^{-\beta t}}$

M 3.16

FOR $b<0$, WE HAVE $\ddot{x} - 2\beta\dot{x} + \omega_0^2 x = 0$

NOTE THE SIGN CHANGE FOR THE $\dot{x}$ TERM.

GEN'L SOLUTION IS

$$x(t) = e^{+\beta t}\left[A_1 \exp[\beta^2-\omega_0^2]^{1/2} t + A_2 \exp -[\beta^2-\omega_0^2]^{1/2} t\right]$$

THE EXPONENTIAL FACTOR $e^{+\beta t}$ GROWS W/O BOUND AS $t \to \infty$.

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$$\ddot{z} + 4\dot{z} + 3z = 0$$

AUXILIARY EQN:

$$r^2 + 4r + 3r = 0$$

$$\Rightarrow r = \frac{-4 \pm [16 - 12]^{1/2}}{2}$$

$$r = -2 \pm 1$$

SO,

$$z(t) = c_1 e^{-3t} + c_2 e^{-t}$$

$$z(0) = 0 \Rightarrow c_1 + c_2 = 0$$

$$\dot{z}(0) = 1 \Rightarrow -3c_1 - c_2 =$$

HENCE, $c_1 = -1/2$, $c_2 = 1/2$

$$\boxed{z(t) = -\tfrac{1}{2} e^{-3t} + \tfrac{1}{2} e^{-t}}$$