HW1 PHYS 3344 E504 HEC

#1

$$x = \frac{\begin{vmatrix} 6 & 2 & 1 \\ 1 & -1 & 2 \\ 0 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & -3 \end{vmatrix}}$$

$$= \frac{6(3-2) - 2(-3-0) + 1(1-0)}{3(3-2) - 2(-3-2) + 1(1-(-1))}$$

$$\boxed{x = 13/15}$$

$$y = \frac{\begin{vmatrix} 3 & 6 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -3 \end{vmatrix}}{15}$$

$$\boxed{y = 20/15 = 4/3}$$

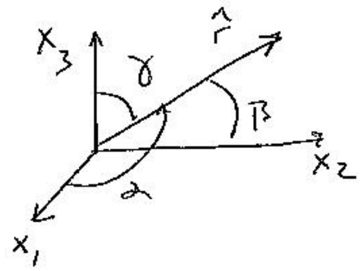
#1. Since $x + y - 3z = 0$

$$z = (x+y)/3$$

$$\boxed{z = 11/15}$$

#2

EQ 1.10 Let $\hat{r}$ = UNIT VECTOR ∥ to DIRECTION OF LINE IN FIGURE



USE $\hat{r}\cdot\hat{r} = 1$ (1)

$$\hat{r} = \hat{r}\cdot\hat{x}_1\,\hat{x}_1 + \hat{r}\cdot\hat{x}_2\,\hat{x}_2 + \hat{r}\cdot\hat{x}_3\,\hat{x}_3 \quad (2)$$

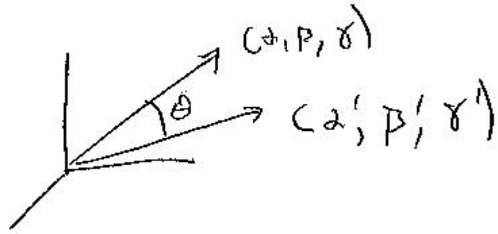
$$= \cos\alpha\,\hat{x}_1 + \cos\beta\,\hat{x}_2 + \cos\gamma\,\hat{x}_3 \quad (3)$$

FROM (1) AND (3)

$$\boxed{\therefore\ \hat{r}\cdot\hat{r} = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

#2.

EQ 1.11 SIMILAR STORY. BUILD 2 UNIT VECTORS



$$\hat{r} = \cos\alpha\ \hat{x}_1 + \cos\beta\ \hat{x}_2 + \cos\gamma\ \hat{x}_3 \quad (1)$$

(RECYCLE FROM ABOVE)

$$\hat{r}' = \cos\alpha'\ \hat{x}_1 + \cos\beta'\ \hat{x}_2 + \cos\gamma'\ \hat{x}_3 \quad (2)$$

$$\hat{r}\cdot\hat{r}' = |\hat{r}|\,|\hat{r}'|\cos\theta = \cos\theta \quad (3)$$

SUB (1) & (2) INTO LHS OF (3):

$$\boxed{\cos\alpha\cos\alpha' + \cos\beta\cos\beta' + \cos\gamma\cos\gamma' = \cos\theta}$$

1-14.

b)

$$\mathbf{AC}=\begin{bmatrix}1 & 2 & -1\\ 0 & 3 & 1\\ 2 & 0 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ 4 & 3\\ 1 & 0\end{bmatrix}=\begin{bmatrix}9 & 7\\ 13 & 9\\ 5 & 2\end{bmatrix}$$

$$\boxed{\mathbf{AC}=\begin{bmatrix}9 & 7\\ 13 & 9\\ 5 & 2\end{bmatrix}}$$

d)

$$\mathbf{AB}-\mathbf{B}'\mathbf{A}'=?$$

$$\mathbf{AB}=\begin{bmatrix}1 & -2 & 1\\ 1 & -2 & 9\\ 5 & 3 & 3\end{bmatrix}\qquad \text{(from part } \mathbf{a}\text{)}$$

$$\mathbf{B}'\mathbf{A}'=\begin{bmatrix}2 & 0 & 1\\ 1 & -1 & 1\\ 0 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & 3 & 0\\ -1 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 5\\ -2 & -2 & 3\\ 1 & 9 & 3\end{bmatrix}$$

$$\boxed{\mathbf{AB}-\mathbf{B}'\mathbf{A}'=\begin{bmatrix}0 & -3 & -4\\ 3 & 0 & 6\\ 4 & -6 & 0\end{bmatrix}}$$

1-15. If **A** is an orthogonal matrix, then

$$\mathbf{A}'\mathbf{A}=\mathbf{1}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & a & a\\ 0 & -a & a\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & a & -a\\ 0 & a & a\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 2a^2 & 0\\ 0 & 0 & 2a^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\cancel{\boxed{a=\frac{1}{\sqrt{2}}}}\quad a=\pm\frac{1}{\sqrt{2}}$$

1-20.

a) Consider the following two cases:

When $i \neq j$ $\quad \delta_{ij} = 0 \quad$ but $\varepsilon_{ijk} \neq 0$.

When $i = j$ $\quad \delta_{ij} \neq 0 \quad$ but $\varepsilon_{ijk} = 0$.

Therefore,

$$\boxed{\sum_{ij} \varepsilon_{ijk}\, \delta_{ij} = 0} \tag{1}$$

b) We proceed in the following way:

When $j = k$, $\varepsilon_{ijk} = \varepsilon_{ijj} = 0$.

Terms such as $\varepsilon_{j11}\, \varepsilon_{l11} = 0$. Then,

$$\sum_{jk} \varepsilon_{ijk}\, \varepsilon_{ljk} = \varepsilon_{i12}\, \varepsilon_{l12} + \varepsilon_{i13}\, \varepsilon_{l13} + \varepsilon_{i21}\, \varepsilon_{l21} + \varepsilon_{i31}\, \varepsilon_{l31} + \varepsilon_{i32}\, \varepsilon_{l32} + \varepsilon_{i23}\, \varepsilon_{l23}$$

Now, suppose $i = l = 1$, then,

$$\sum_{jk} = \varepsilon_{123}\, \varepsilon_{123} + \varepsilon_{132}\, \varepsilon_{132} = 1 + 1 = 2$$

for $i = l = 2$, $\sum\limits_{jk} = \varepsilon_{213}\, \varepsilon_{213} + \varepsilon_{231}\, \varepsilon_{231} = 1 + 1 = 2$. For $i = l = 3$, $\sum\limits_{jk} = \varepsilon_{312}\, \varepsilon_{312} + \varepsilon_{321}\, \varepsilon_{321} = 2$. But $i = 1$, $l = 2$ gives $\sum\limits_{jk} = 0$. Likewise for $i = 2$, $l = 1$; $i = 1$, $l = 3$; $i = 3$, $l = 1$; $i = 2$, $l = 3$; $i = 3$, $l = 2$.

Therefore,

$$\boxed{\sum_{j,k} \varepsilon_{ijk}\, \varepsilon_{ljk} = 2\delta_{il}} \tag{2}$$

c) $$\sum_{ijk} \varepsilon_{ijk}\, \varepsilon_{ijk} = \varepsilon_{123}\, \varepsilon_{123} + \varepsilon_{312}\, \varepsilon_{312} + \varepsilon_{321}\, \varepsilon_{321} + \varepsilon_{132}\, \varepsilon_{132} + \varepsilon_{213}\, \varepsilon_{213} + \varepsilon_{231}\, \varepsilon_{231}$$

$$= 1 \cdot 1 + 1 \cdot 1 + (-1) \cdot (-1) + (-1) \cdot (-1) + (-1) \cdot (-1) + (1) \cdot (1)$$

or,

$$\boxed{\sum_{ijk} \varepsilon_{ijk}\, \varepsilon_{ijk} = 6} \tag{3}$$