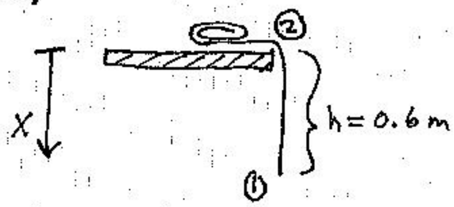
M 2-27



$m = 0.4\ kg$
$\ell = 4\ m$
$\rho = m/\ell = 0.1\ kg/m$
$g \simeq 10\ m/s^2$

WORK DONE BY FORCE $= W_{12} = \int_1^2 mg\,dx$

$$= \int_1^2 (\rho x)\,g\,dx = \int_1^2 \rho g\,x\,dx$$

$$W_{12} = \rho g \frac{x^2}{2}\Big|_0^{0.6}$$

$$= \frac{(0.1)\,10\,(0.36)}{2}\ J$$

$$\boxed{W_{12} = 0.18\ J}$$

NOTE THAT $W_{12} > 0$. IT TAKES ENERGY TO LIFT THE ROPE ONTO THE TABLE.

PHYS 3344 HW Sols TEC

#1 $M = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$

$$M^{-1} = \frac{M_c^T}{\|M\|}$$

$$\|M\| = 1(-1) - 2(-2) + 1(-2)$$
$$= 1$$

$$M_c = \begin{bmatrix} -1 & 2 & -2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} ;$$

$$M_c^T = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & -1 \\ -2 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \boxed{M^{-1} = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & -1 \\ -2 & 3 & 1 \end{bmatrix}}$$

FERMI #2

$$V_T = \#\text{BREATHS} \times \text{Vol}/\text{BREATH}$$

$$= \left(\#\text{BR}/\text{UNIT-TIME}\right)\left(\text{LIFESPAN}\right) * \text{Vol}/\text{BR}$$

$$\simeq \left(10/\text{MIN}\right)\left(75\pi \times 10^7 \text{ sec}\right) * 60^{-1} \text{ min/sec} * 1 \text{ liter}$$

$$\boxed{V_T \simeq 4 \times 10^8 \text{ liters}}$$

M 2-11

$$F_{NET} = mg - kmv^2$$

$$m\frac{dv}{dt} = mg - kmv^2$$

$$\frac{dv}{dx}\frac{dx}{dt} = g(1 - c_0^2 v^2), \qquad c_0^2 \equiv k/g$$

$$\frac{v\,dv}{1 - c_0^2 v^2} = g\,dx$$

$$\Rightarrow -\frac{1}{2c_0^2}\ln(1 - c_0^2 v^2) = gx + \cancelto{0}{const} \quad (x = v = 0 @ t=$$

$$\boxed{x = -\frac{1}{2c_0^2 g}\ln(1 - c_0^2 v^2)}$$

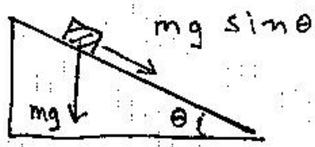
so,

$$x_1 - x_0 = -\frac{1}{2c_0^2 g}\left[\ln(1 - c_0^2 v_1^2) - \ln(1 - c_0^2 v_0^2)\right]$$

$$= +\frac{1}{2c_0^2 g}\ln\left[\frac{1 - c_0^2 v_0^2}{1 - c_0^2 v_1^2}\right]$$

$$\boxed{x_1 - x_0 = \frac{1}{2k}\ln\left[\frac{g - kv_0^2}{g - kv_1^2}\right]}$$

M 2-15



PROBLEM SIMILAR TO 2-11.

$$m\frac{dv}{dt} = mg\sin\theta\,(1 - c_0^2 v^2), \qquad c_0^2 = k/g\sin\theta$$

$$\frac{dv}{1-c_0^2 v^2} = g\sin\theta\, dt$$

$$\frac{dz}{1-z^2} = c_0 g\sin\theta\, dt, \qquad z \equiv c_0 v$$

$$\frac{1}{2}\ln\frac{1+z}{1-z} = c_0 g\sin\theta\, t + \cancel{const}^{\,0} \;;\; z=0 \text{ @ } t=0$$

$$\frac{1+z}{1-z} = e^{2c_0 g\sin\theta\, t}$$

$$z = \frac{e^{2c_0 g\sin\theta\, t} - 1}{e^{2c_0 g\sin\theta\, t} + 1} = \frac{e^{c_0 g\sin\theta\, t} - e^{-c_0 g\sin\theta\, t}}{e^{c_0 g\sin\theta\, t} + e^{-c_0 g\sin\theta\, t}}$$

$$c_0\frac{dx}{dt} = \tanh c_0 g\sin\theta\, t$$

$$\int dx = \frac{1}{c_0^2 g\sin\theta}\int \tanh s\, ds, \qquad s \equiv c_0 g\sin\theta\, t$$

$$\& \; s=0 \text{ @ } t=0$$

$$x = \frac{1}{c_0^2 g\sin\theta}\ln\cosh s$$

$$e^{c_0^2 g\sin\theta\, d} = \cosh c_0 g\sin\theta\, t$$

$$\therefore \boxed{t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg\sin\theta}}}$$

M 2-25.

(a) $F_N = \frac{mv^2}{r}$, NET FORCE ON m.

$$\underset{\sim}{F}_N = \underset{\sim}{F}_{TRACK} + \underset{\sim}{F}_g \Rightarrow F_T = F_N + mg$$

AT PT A:

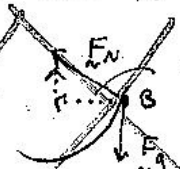
$$\tfrac{1}{2} mv^2 = mgh \qquad \text{CONS. OF ENERGY}$$

$$v^2 = 2gh$$

So,

$$F_N = \frac{2mgh}{R} \Rightarrow \boxed{\underset{\sim}{F}_T = mg\left(\frac{2h}{R} + 1\right), \text{ POINTING UP}}$$

(b) m IS EXECUTING CIRCULAR MOTION DUE TO FORCE THAT TRACK EXERTS ON IT.



$\theta = 45°$

AGAIN,

$$\underset{\sim}{F}_{NET} = \underset{\sim}{F}_T + \underset{\sim}{F}_g$$

Y DIR: $F_N^Y = \frac{mv^2}{R} \sin 45° = F_T^Y + mg$

X DIR: $F_N^X = \frac{mv^2}{R} \cos 45° = F_T^X$

AT PT B BY CONS. OF ENERGY: $\tfrac{1}{2} mv^2 + mgR(1-\cos 45°) = mgh$

$$v^2 = 2g\left(h - \frac{\sqrt{2}-1}{\sqrt{2}} R\right)$$

$\} h' = R(1-\cos\alpha)$

So, $|\underset{\sim}{F}_T| = \left[(F_T^X)^2 + (F_T^Y)^2\right]^{1/2}$

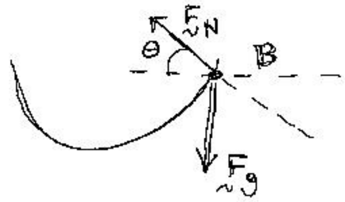
$$= \left[\left(\frac{mv^2}{R}\right)^2 + (mg)^2\right]^{1/2}$$

$$= \left[\frac{m^2}{R^2}\left(2g\left(h - \frac{\sqrt{2}-1}{\sqrt{2}} R\right)\right)^2 + m^2 g^2\right]^{1/2}$$

$$\therefore \boxed{F_T = mg\left[\left(2\left(\frac{h}{R} - \frac{\sqrt{2}-1}{\sqrt{2}}\right)\right)^2 + 1\right]^{1/2}}$$

(c) FROM (b) $\boxed{v_B = \left[2gh\left(1 - \frac{\sqrt{2}-1}{\sqrt{2}} \frac{R}{h}\right)\right]^{1/2}}$

b) m IS EXECUTING CIRCULAR MOTION DUE TO FORCE THAT TRACK EXERTS ON IT.



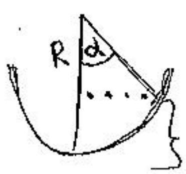
$\theta = 45°$

$\underset{\sim}{F}_N = \underset{\sim}{F}_T + \underset{\sim}{F}_g$; $\underset{\sim}{F}_T$ = FORCE OF TRACK ON

$$\Rightarrow F_T = \frac{mv^2}{R} + mg\cos\theta$$

$$= \frac{mv^2}{R} + \frac{mg}{\sqrt{2}} . \qquad (10)$$

AT PT B BY CONS. OF ENERGY: $\frac{1}{2}mv^2 + mgR(1-\cos 45°) = mg$



$h' = R(1-\cos\alpha)$

$$v^2 = 2g\left(h - \frac{\sqrt{2}-1}{\sqrt{2}}R\right)$$

USING EQ.(10)

$$F_T = \frac{2mg}{R}\left(h - \frac{\sqrt{2}-1}{\sqrt{2}}R\right) + \frac{mg}{\sqrt{2}}$$

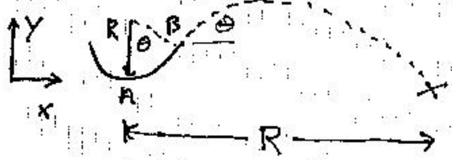
$$= 2mg\frac{h}{R} - 2mg + 3\frac{mg}{\sqrt{2}}$$

$$\therefore \boxed{F_T = mg\left[2\frac{h}{R} + \frac{3-2\sqrt{2}}{\sqrt{2}}\right]}$$

(c) FROM (b)

$$\boxed{v_B = \left[2gh\left(1 - \frac{\sqrt{2}-1}{\sqrt{2}}\frac{R}{h}\right)\right]^{1/2}}$$

(d) THIS IS REALLY JUST PROJECTILE MOTION.



$\theta = 45°$

$$y = y_0 + V_B \sin\theta\, t - \frac{1}{2} g t^2$$

$$0 = \frac{\sqrt{2}-1}{\sqrt{2}} R + V_B \sin\theta\, t - \frac{1}{2} g t^2$$

$$\Rightarrow t = \frac{\frac{V_B}{\sqrt{2}} + \left[\frac{V_B^2}{2} + 2g\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) R \right]^{1/2}}{g}, \quad t = \text{FLIGHT TIME}$$

ONLY + SIGN HAS PHYSICAL MEANING HERE $(t > 0)$

$$x = R \sin\theta + V_B \cos\theta \cdot t$$

$$= \frac{R}{\sqrt{2}} + \frac{\frac{V_B}{\sqrt{2}} \left[\frac{V_B}{\sqrt{2}} + \left[\frac{V_B^2}{2} + 2g \frac{\sqrt{2}-1}{\sqrt{2}} R \right]^{1/2} \right]}{g}$$

FROM PT B, $v_B^2 = 2g\left(h - \frac{\sqrt{2}-1}{\sqrt{2}} R\right)$,

SO,

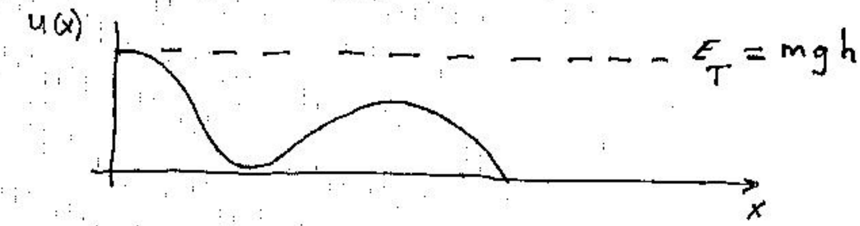
$$x = \frac{R}{\sqrt{2}} + \left(h - \frac{\sqrt{2}-1}{\sqrt{2}} R\right) + \left[h^2 - \sqrt{2}(\sqrt{2}-1) R h + \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 R^2 + \sqrt{2}(\sqrt{2}-1) R h - (\sqrt{2}-1)^2 R^2 \right]^{1/2}$$

$$= \frac{R}{\sqrt{2}} + \left(h - \frac{\sqrt{2}-1}{\sqrt{2}} R\right) + \left[h^2 - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 R^2 \right]^{1/2}$$

$$\boxed{X = h + (\sqrt{2}-1) R + h \left[1 - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)^2 \frac{R^2}{h^2} \right]^{1/2}}$$

M-25

PT (e) $U(x) = mg\,y(x)$, so $U(x)$ is proportional to y.



M 2-50

(a) BY HYPOTHESIS, $\dot{m} = kA = k\pi r^2$

BUT, $m = \rho \frac{4}{3}\pi r^3 \Rightarrow \dot{m} = 4\pi r^2 \rho \dot{r}$

MASS DENSITY

$k\pi r^2 = 4\pi r^2 \rho \dot{r}$

$\dot{r} = k/4\rho$

$$\boxed{r = \frac{kt}{4\rho} + r_0}$$

(b) NEWTON'S 2ND LAW: $\frac{d}{dt}(mv) = mg$

$$\frac{4}{3}\pi\rho \frac{d}{dt}(r^3 v) = \frac{4}{3}\pi\rho r^3 g$$

$$d(r^3 v) = g r^3 dt = \frac{g k^3}{4^3 \rho^3} t^3 dt \quad \text{(ASSUME } r_0 \simeq 0\text{)}$$

NTEGRATE:

$$r^3 v = \frac{g k^3}{4^4 \rho^3} t^4 \quad (r = 0 \text{ @ } t = 0)$$

BUT, $t = \frac{4\rho r}{k} \Rightarrow r^3 v = \frac{g k^3}{4^4 \rho^3} \cdot \frac{4^4 \rho^4 r^4}{k^4}$

SO,

$$v = \frac{g\rho}{k} r = \frac{g\rho}{k} \cdot \frac{kt}{4\rho}$$

$$\therefore \boxed{v = \frac{g}{4} t}$$