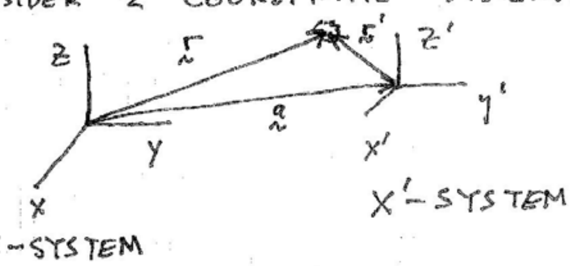
PHYS3344 SOLUTIONS

M9-13

NET TORQUE $\underset{\sim}{N}_N = \sum_\alpha \underset{\sim}{r}_\alpha \times \underset{\sim}{F}_\alpha$

FOR THIS PROBLEM $\sum_\alpha \underset{\sim}{F}_\alpha = 0$, NET FORCE ON SYSTEM $= 0$

CONSIDER 2 COORDINATE SYSTEMS:



$\underset{\sim}{r}_\alpha' = \underset{\sim}{a} + \underset{\sim}{r}_\alpha'$

FOR EACH PARTICLE α

NOW,

$\underset{\sim}{N}_N$ (wrt X ORIGIN) $= \sum_\alpha \underset{\sim}{r}_\alpha \times \underset{\sim}{F}_\alpha$

$\underset{\sim}{N}_N$ (wrt X' ORIGIN) $= \sum_\alpha \underset{\sim}{r}_\alpha' \times \underset{\sim}{F}_\alpha$

$$= \sum_\alpha (\underset{\sim}{a} + \underset{\sim}{r}_\alpha) \times \underset{\sim}{F}_\alpha$$

$$= \sum_\alpha (\underset{\sim}{a} \times \underset{\sim}{F}_\alpha) + (\underset{\sim}{r}_\alpha \times \underset{\sim}{F}_\alpha)$$

$$= \underset{\sim}{a} \times \underbrace{\sum_\alpha \underset{\sim}{F}_\alpha}_{0} + \sum_\alpha \underset{\sim}{r}_\alpha \times \underset{\sim}{F}_\alpha$$

$$= \sum_\alpha \underset{\sim}{r}_\alpha \times \underset{\sim}{F}_\alpha$$

$= \underset{\sim}{N}_N$ (wrt X ORIGIN)

SINCE $\underset{\sim}{a}$ IS ARBITRARY, WORKS FOR ANY 2 COORDINATE SYSTEMS AS LONG AS $\sum_\alpha \underset{\sim}{F}_\alpha = 0$.

M 9-24 TRICKY PROBLEM. ALTHOUGH IT MAY APPEAR SO, ANGULAR MOMENTUM $\underset{\sim}{L}$ OF m IS NOT CONSERVED. THE REASON IS THAT THE FORCE BETWEEN m AND THE CYLINDER IS NOT BETWEEN THEIR INDIVIDUAL COM'S. ALTHOUGH THE FORCE m EXERTS ON THE CYLINDER HAS THE SAME MAGNITUDE AS THE FORCE THE CYLINDER EXERTS ON m, THIS IS NOT ENOUGH TO GUARANTEE $\underset{\sim}{L} = \text{const.}$

WHAT IS CONSERVED IS KE OF THE SYSTEM SINCE THERE ARE NO EXTERNAL FORCES. (YOU ARE TOLD TO IGNORE GRAVITY.)

SO,

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2$$

$$v^2 = v_0^2$$

$$\omega r = \omega_0 r_0 \, , \quad r = \text{DISTANCE BETWEEN } m \text{ \& CYLINDER}$$

BUT AS STRING WRAPS AROUND CYLINDER, r SHRINKS. r SHRINKS BY $s = a\theta$. HENCE

$$\boxed{\omega = \frac{\omega_0 r_0}{r} = \frac{\omega_0 b}{b - a\theta} = \omega_0 \frac{1}{(1 - \frac{a}{b}\theta)}}$$

m MOVES IN CIRCLE OF CONSTANTLY CHANGING RADIUS. (OVERALL EFFECT IS TO TRAVEL IN SPIRAL.)

CENTRIPETAL FORCE ON m: $F_c = m \omega^2 r$

$$= \frac{m \omega_0^2}{(1 - \frac{a}{b}\theta)^2} \cdot (b - a\theta)$$

$$= \frac{m \omega_0}{(1 - \frac{a}{b}\theta)} b \, \omega_0$$

$$\therefore \boxed{F_c = m \omega \omega_0 b}$$

M 9-28

KE = CONST (ELASTIC COLLISION)

IN GEN'L $\dfrac{T_1}{T_0} = 1 - \dfrac{2m_1 m_2}{(m_1+m_2)^2}(1-\cos\theta)$ EQ. 9.87a

NOW, THE FRACTION OF KE LOST IS $1 - T_1/T_0$.

TO <u>MAXIMIZE</u> THIS FRACTION, WE NEED TO <u>MINIMIZE</u> T_1/T_0. HENCE, WE WANT TO <u>MAXIMIZE</u> $(1-\cos\theta)$. THIS OCCURS WHEN $\cos\theta = -1$. WE HAVE

$$\left(T_1/T_0\right)_{MIN} = 1 - \frac{2m_1 m_2}{(m_1+m_2)^2} * 2$$

$$= 1 - \frac{4m_1 m_2}{(m_1+m_2)^2}$$

$$\Rightarrow \left(1 - T_1/T_0\right)_{MAX} = 1 - \left(1 - \frac{4m_1 m_2}{(m_1+m_2)^2}\right)$$

$$\boxed{\left(1 - T_1/T_0\right)_{MAX} = \frac{4m_1 m_2}{(m_1+m_2)^2}}$$

PHYS 3344 SOLUTIONS

5-3. GRAVITATIONAL FORCE IS CONSERVATIVE

SO $E_{TOT} = \text{CONST.} = U + T$

AT ∞, $E\Big|_{MIN} = U_\infty + T_\infty\Big|_{MIN} = 0 + 0$

AT EARTH'S SURFACE,

$$E = -\frac{GMm}{R_E} + \frac{1}{2} m V_{ESC}^2 = 0$$

$$\Rightarrow \boxed{V_{ESC} = \sqrt{\frac{2GM_E}{R_E}}}$$

$G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$

$M_E = 5.97 \times 10^{24}\ \text{KG}$

$R_E = 6.38 \times 10^{6}\ \text{M}$

NUMERICALLY,

$$\boxed{V_{ESC} \simeq 11\ \text{km/s}.}$$

5-5

$$F = ma = m v \frac{dv}{dR} = -\frac{G M_E m}{R^2}$$

$$\Rightarrow \int v\,dv = -\int \frac{G M_E}{R^2}\,dR$$

$$\left.\frac{v^2}{2}\right|_0^{v'} = \left.\frac{GM}{R}\right|_{R_0}^{R'}$$

LET $R_0 \to \infty$ SO THAT $v = -\sqrt{2GM_E}\; R^{-1/2}$

$$v = \frac{dR}{dt} = -\sqrt{2GM_E}\; R^{-1/2}$$

$$R^{1/2}\,dR = -\sqrt{2GM_E}\; dt$$

$$-\frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM_E}} = t + C_0, \qquad R = R_0 \text{ @ } t = 0$$

$$\Rightarrow t = \frac{2}{3}\frac{R_0^{3/2}}{\sqrt{2GM_E}} - \frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM_E}}$$

$$T_{FULL} = \frac{2}{3}\frac{R_0^{3/2}}{\sqrt{2GM_E}}, \qquad T_{1/2} = \frac{2}{3\sqrt{\;}} R_0^{3/2}\left(1 - \left(\tfrac{1}{2}\right)^{3/2}\right)$$

(T_{FULL} OCCURS WHEN $R = 0$, $T_{1/2} \sim R = \frac{R_0}{2}$)

$$T_{1/2}/T_{FULL} = \frac{1 - \left(\frac{1}{2}\right)^{3/2}}{1} \simeq 0.65$$

$$\therefore \boxed{T_{1/2}/T_{FULL} \simeq 7/11}$$