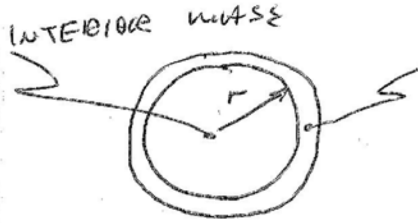
5.14

CONSIDER SPHERE AS MADE FROM A COLLECTION OF SHELLS



ρ = MASS DENSITY

R = SPHERE RADIUS

$$dU = -\frac{G M_{INT} M_{SHELL}}{r}$$

$$dU = -\frac{G\left(\frac{4}{3}\pi\rho r^3\right)4\pi r^2 \rho\, dr}{r}$$

$$= -G\frac{16}{3}\pi^2\rho^2 r^4 dr$$

$$U = \int_0^R dU = -\int_{r=0}^{R} G\frac{16}{3}\pi^2\rho^2 r^4 dr$$

$$= -G\left(\frac{16}{3}\right)\pi^2\rho^2 R^5/5 \qquad (1)$$

BUT $\rho = M/\frac{4}{3}\pi R^3 \Rightarrow \rho^2 = \frac{9}{16\pi^2}\frac{M^2}{R^6}$

SO,

$$\boxed{U = -\frac{3}{5}GM^2/R}$$

8-3

VALUE of ε DETERMINES BASIC ORBIT SHAPE

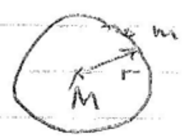
$$\varepsilon = \left[1 + \frac{2El^2}{\mu k^2}\right]^{1/2} \qquad (1)$$

FOR PROBLEM AT HAND, $l = \text{CONST}$ BECAUSE FORCE IS ALWAYS CENTRAL. GRAV. PE $\neq$ CONST BECAUSE FORCE LAW CHANGES. CONNECTION BETWEEN THE TWO IS

$$F_G = -k'/r^2$$

$$U_G = -k'/r$$

NEED TO CALCULATE NEW VALUE OF E AFTER TRANSFORMATION. START W/ ORIGINAL SITUATION:



$$F_G = \frac{GMm}{r^2} = mv^2/r$$

$$\Rightarrow v^2 = GM/r$$

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\frac{GMm}{r}$$

8-3

$$E_{TOT} = KE + GPE$$

$$= \frac{1}{2}mv^2 - GMm/r$$

$$= \frac{1}{2}\frac{GMm}{r} - GMm/r$$

$$E_{TOT} = -\frac{1}{2}GMm/r = -k/2r$$

KE FIXED since l = const ~~[illegible]~~ AND OBJECT MOVING IN CIRCLE @ MOMENT OF XFORM.

$$KE = \frac{1}{2}GMm/r$$

$$U_{NEW} = \frac{-k}{2r} \quad (\text{since } k \to k/2)$$

$$= -\frac{GMm}{2r}$$

$$E_{TOT}\Big|_{NEW} = KE + GPE = \frac{GMm}{2r} - \frac{GMm}{2r} = 0$$

FROM EQ (1), $E = 0 \Rightarrow \varepsilon = 1$

$\therefore$ ORBIT IS A PARABOLA

M 8.6

$t=0$:

$$E = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\frac{\ell^2}{\mu r^2} - \frac{k}{r} = \text{CONST} = E_0$$

OBJECTS RELEASED FROM REST $\Rightarrow \ell = 0$, $\dot{r} = 0$

NOW, $E_0 = -k/r_0$

AFTER TIME t

$$E_0 = -\frac{k}{r_0} = \frac{1}{2}\mu \dot{r}^2 - k/r$$

$$\Rightarrow \dot{r} = \left[\frac{2}{\mu} k \left(\frac{1}{r} - \frac{1}{r_0}\right)\right]^{1/2} \qquad (1)$$

BUT, $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{m_1 m_2}{M}$

AND $v_1 = \dfrac{m_2 \dot{r}}{m_1 + m_2}$ & $v_2 = \dfrac{m_1 \dot{r}}{m_1 + m_2}$

$$k = G m_1 m_2$$

HENCE,
$$v_1 = \frac{m_2}{M}\left[2GM\left(\frac{1}{r} - \frac{1}{r_0}\right)\right]^{1/2}$$

$$\boxed{v_1 = m_2 \left[\frac{2G}{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right]^{1/2}}$$

$$\boxed{v_2 = m_1 \left[\frac{2G}{M}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right]^{1/2}}$$

M8.14

RECALL EQN 8.21, p297

$$\frac{d^2}{d^2\theta}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{\ell^2} F(r) \qquad (1)$$

Now, $r = k\theta^2$

$$\frac{d}{d\theta}\left(\frac{\theta^{-2}}{k}\right) = -2\theta^{-3}/k$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = \frac{d}{d\theta}\left(-2\theta^{-3}/k\right) = +\frac{6\theta^{-4}}{k}$$

SUB. INTO (1) YIELDS

$$6/k\theta^4 + \frac{1}{k\theta^2} = -\frac{\mu r^2}{\ell^2} F(r)$$

$$\Rightarrow F(r) = -\frac{\ell^2}{\mu}\frac{1}{k^2\theta^4}\left[\frac{6+\theta^2}{k\theta^4}\right]$$

$$= -\frac{\ell^2}{\mu}\left[\frac{6}{k^3\theta^8} + \frac{1}{k^3\theta^6}\right]$$

BUT $\theta^2 = r/k$, SO

$$F(r) = -\frac{\ell^2}{\mu}\left[\frac{6}{k^3}\frac{k^4}{r^4} + \frac{1}{k^3}\frac{k^3}{r^3}\right]$$

$$\boxed{F(r) = -\frac{\ell}{\mu}\left[\frac{6k}{r^4} + \frac{1}{r^3}\right]}$$

$$l = \mu r^2 \dot{\theta} = \text{CONST} \qquad (1)$$

$$\frac{r_{MAX}}{r_{MIN}} = \frac{1+\varepsilon}{1-\varepsilon} \qquad (2)$$

FROM (1):

$$\mu r_{MIN}^2 \dot{\theta}_{MAX} = \mu r_{MAX}^2 \dot{\theta}_{MIN}$$

$$\Rightarrow \frac{\dot{\theta}_{MAX}}{\dot{\theta}_{MIN}} = n = \frac{r_{MAX}^2}{r_{MIN}^2}$$

SUB INTO (2):

$$\sqrt{n} = \frac{1+\varepsilon}{1-\varepsilon}$$

$$(1-\varepsilon)\sqrt{n} = 1+\varepsilon$$

$$\sqrt{n} - 1 = \varepsilon(1+\sqrt{n})$$

$$\boxed{\varepsilon = \frac{\sqrt{n}-1}{\sqrt{n}+1}}$$