PHYS 3344 SOLUTIONS

M 6.2

$$J = \int ds = \int \sqrt{dx^2 + dy^2} = \int [1 + y'^2]^{1/2} dx$$

WHERE ds = ELEMENT OF PATH LENGTH

EULER'S EQN: $$\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$ X = INDEPENDENT VARIABLE

NOW
$$f = [1 + y'^2]^{1/2}$$
AND
$$\frac{\partial f}{\partial y} = 0, \quad \text{SO} \quad \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$$

$$\frac{\partial f}{\partial y'} = C_0, \qquad C_0 = \text{CONST.}$$

$$\frac{1}{2}[1 + y'^2]^{-1/2}\, 2y' = C_0$$

$$\frac{y'}{[1+y'^2]^{1/2}} = C_0$$

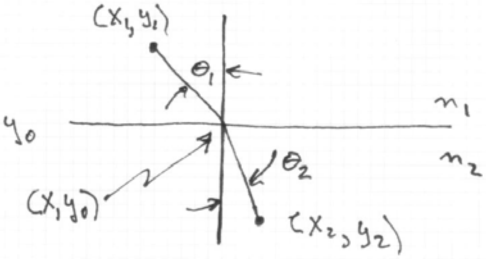
$$y'^2 = C_0^2 (1 + y'^2)$$

$$y'^2 (1 - C_0^2) = C_0^2$$

$$y' = \sqrt{\frac{C_0^2}{1 - C_0^2}} \equiv C_1$$

$$\therefore \boxed{y = C_1 x + C_2} \qquad C_2 = \text{CONST}$$

M 6.7



$n_2 > n_1$

TOTAL TRANSIT TIME $T = \dfrac{\sqrt{(x_1 - x)^2 + (y_1 - y_0)^2}}{c/n_1} + \dfrac{\sqrt{(x - x_2)^2 + (y_0 - y_2)^2}}{c/n_2}$

$y = y_0$ IS BOUNDARY
BETWEEN n_1 & n_2 MATERIAL

~~XE THEE~~
FERMAT'S PRINCIPLE : MINIMIZE TOTAL TRAVEL TIME T.

$$\frac{dT}{dx} = 0 = \frac{\frac{1}{2}\left[(x_1 - x)^2 + (y_1 - y_0)^2\right]^{-1/2} 2(x_1 - x)(-1)}{c/n_1} + \frac{\frac{1}{2}\left[(x - x_2)^2 + (y_0 - y_2)^2\right]^{-1/2} 2(x - x_2)}{c/n_2}$$

$$\frac{n_1 (x_1 - x)}{\left[(x_1 - x)^2 + (y_1 - y_0)^2\right]^{1/2}} = \frac{n_2 (x - x_2)}{\left[(x - x_2)^2 + (y_0 - y_2)^2\right]^{1/2}}$$

FROM GEOMETRY OF FIGURE

$$\therefore \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

NOTE THAT (x, y_0) IS INTERSECTION PT W/ BOUNDARY.

FERMI PROBLEM

ESTIMATE MASS OF HUMAN BODY.
THEN CALCULATE # OF ATOMS.

$$\#\text{ATOMS} \simeq M\_\text{BODY} * \left(\frac{\text{MOLE}}{\text{UNIT MASS}}\right) * \left(\frac{\#\text{ATOMS}}{\text{MOLE}}\right)$$

$$M\_\text{BODY} \simeq 100^{\#} = 4.54 \times 10^{4}\ \text{gm}$$

HUMAN BODY MOSTLY CARBON (^{12}C)

$$\#\text{ATOMS} \simeq 4.54 \times 10^{4}\ \text{gm} \left(\frac{1\ \text{MOLE}}{12\ \text{gm}}\right) * \left(\frac{6.02 \times 10^{23}\ \text{ATOMS}}{\text{MOLE}}\right)$$

$$\boxed{N \simeq 2 \times 10^{27}\ \text{ATOMS}}$$