M7.3

$$T = \frac{1}{2} m (R-\rho)^2 \dot\theta^2 + \frac{1}{2} I \dot\varphi^2 \qquad , I = \frac{2}{5} m \rho^2 \text{ (SPHERE)}$$

$$= \frac{1}{2} m (R-\rho)^2 \dot\theta^2 + \frac{1}{5} m \rho^2 \dot\varphi^2$$

$$U = mg(R-\rho)(1-\cos\theta)$$

$\boxed{f = R\theta - \rho\varphi = 0}$ SPHERE ROLLS w/o SLIPPING.

$$\boxed{L = \frac{1}{2} m (R-\rho)^2 \dot\theta^2 + \frac{1}{5} m \rho^2 \dot\varphi^2 - mg(R-\rho)(1-\cos\theta)}$$

$$\frac{\partial L}{\partial \theta} = -mg(R-\rho)\sin\theta$$

$$\frac{\partial L}{\partial \dot\theta} = m(R-\rho)^2 \dot\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = m(R-\rho)^2 \ddot\theta$$

$$\frac{\partial f}{\partial \theta} = R$$

$$\Rightarrow \boxed{-mg(R-\rho)\sin\theta - m(R-\rho)^2\ddot\theta + \lambda R = 0} \quad (1)$$

$$\frac{\partial L}{\partial \varphi} = 0$$

$$\frac{\partial L}{\partial \dot\varphi} = \frac{2}{5} m \rho^2 \dot\varphi$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = \frac{2}{5} m \rho^2 \ddot\varphi$$

$$\frac{\partial f}{\partial \varphi} = -\rho$$

$$\Rightarrow \boxed{-\frac{2}{5} m \rho^2 \ddot\varphi - \lambda\rho = 0}$$

$$\lambda = -\frac{2}{5} m \rho \ddot\varphi$$

$$= -\frac{2}{5} m R \ddot\theta \quad (2)$$

$(R\ddot\theta = \rho\ddot\varphi)$.

3

M7.3 CON'T

FROM EQN (1) ABOVE & USING (2)

$$-mg(R-\rho)\sin\theta - m(R-\rho)^2\ddot{\theta} - \frac{2}{5}mR^2\ddot{\varphi} = 0$$

$$-(R-\rho)g\sin\theta - \left[(R-\rho)^2 + \frac{2}{5}R^2\right]\ddot{\theta} = 0 \qquad (3)$$

FOR SMALL OSCILLATIONS, $\sin\theta \simeq \theta$, SO (3) BECOMES

$$+(R-\rho)g\,\theta + \left[(R-\rho)^2 + \frac{2}{5}R^2\right]\ddot{\theta} = 0$$

NOTE ANALOGY W/ SHM OF MASS ON SPRING:

$$m\ddot{x} + kx = 0$$

$$\nu = \frac{1}{2\pi}\sqrt{k/m}$$

HENCE

$$\boxed{\nu = \frac{1}{2\pi}\left[\frac{(R-\rho)g}{(R-\rho)^2 + \frac{2}{5}R^2}\right]^{1/2}}$$

7.4

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2)$$

$$F = -Ar^{\alpha-1} = -\nabla U \Rightarrow U = \frac{A}{\alpha}r^\alpha$$

$$L = T - U = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{A}{\alpha}r^\alpha$$

$$\frac{\partial L}{\partial r} = -Ar^{\alpha-1} + mr\dot{\theta}^2$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{r}} = m\ddot{r}$$

$$\Rightarrow \boxed{m\ddot{r} - mr\dot{\theta}^2 + Ar^{\alpha-1} = 0} \qquad (1)$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta}$$

$$\Rightarrow \frac{d}{dt}(mr^2\dot{\theta}) = 0 \qquad (2)$$

BUT $\ell = mr^2\dot{\theta} = \measuredangle$ momentum
so $\ell =$ const

AND, (2) $\Rightarrow$ $\boxed{2m r\dot{r}\dot{\theta} + mr^2\ddot{\theta} = 0}$

7.4.

Q: IS $E = \text{CONST}$?

BY ANALOGY w/ GRAVITATIONAL FORCE w/ $F \propto 1/r^2$,

A: $\boxed{\text{YES}}$

EXPLICITLY, REWRITE (1)

$$m\ddot{r} - \frac{\ell^2}{mr^3} + Ar^{\alpha-1} = 0$$

$$m\dot{r}\ddot{r} - \frac{\dot{r}\ell^2}{mr^3} + A\dot{r}r^{\alpha-1} = 0$$

$$\frac{d}{dt}\left(\frac{m}{2}\dot{r}^2\right) + \frac{d}{dt}\left(\frac{\ell^2}{2mr^2}\right) + \frac{d}{dt}\left(Ar^{\alpha}\right) = 0$$

FROM OUR DISCUSSION OF CENTRAL FORCE MOTION,

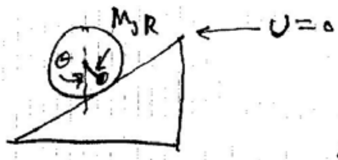
$$KE = \frac{m}{2}\dot{r}^2 + \frac{\ell^2}{2mr^2}$$

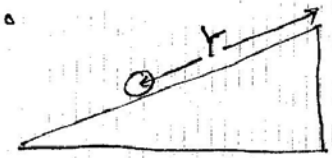
SO,

$$\frac{d}{dt}(KE) + \frac{d}{dt}(U) = 0$$

$$\Rightarrow \boxed{E_{TTL} = KE + U = \text{CONST.}}$$

M 7.9

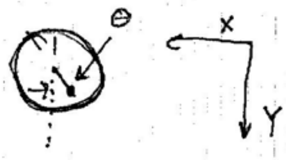




$T = \text{DISK} + \text{PENDULUM}$

$= \frac{1}{2}M\dot{Y}^2 + \frac{1}{4}M\dot{Y}^2 + \text{PENDULUM}$

PENDULUM:



$X(\text{BOB}) = Y\cos\alpha - l\sin\theta$

$Y(\text{BOB}) = Y\sin\alpha + l\cos\theta$

$\dot{x} = \dot{Y}\cos\alpha - l\dot{\theta}\cos\theta$

$\dot{y} = \dot{Y}\sin\alpha - l\dot{\theta}\sin\theta$

$\dot{x}^2 = \dot{Y}^2\cos^2\alpha + l^2\dot{\theta}^2\cos^2\theta - 2\dot{Y}l\dot{\theta}\cos\theta\cos\alpha$

$\dot{y}^2 = \dot{Y}^2\sin^2\alpha + l^2\dot{\theta}^2\sin^2\theta - 2\dot{Y}l\dot{\theta}\sin\alpha\sin\theta$

$\dot{x}^2 + \dot{y}^2 = \dot{Y}^2 + l^2\dot{\theta}^2 - 2\dot{Y}l\dot{\theta}\underbrace{(\sin\alpha\cos\theta + \cos\alpha\sin\theta)}_{\sin(\alpha+\theta)}$

$T = \frac{1}{2}M\dot{Y}^2 + \frac{1}{4}M\dot{Y}^2 + \frac{1}{2}m\dot{Y}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - m\dot{Y}l\dot{\theta}\sin(\alpha+\theta)$

$U = -MgY\sin\alpha - mgl\cos\theta - mgY\sin\alpha$

$$\boxed{\begin{aligned} L = {}& \frac{3}{4}M\dot{Y}^2 + \frac{1}{2}m\dot{Y}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - m\dot{Y}l\dot{\theta}\sin(\alpha+\theta) \\ & + MgY\sin\alpha + mgY\sin\alpha + mgl\cos\theta \end{aligned}}$$

M 7.9

$$\frac{\partial L}{\partial Y} = Mg \sin\alpha + mg \sin\alpha$$

$$\frac{\partial L}{\partial \dot{Y}} = \frac{3}{2} M\dot{Y} + m\dot{Y} - m l \dot{\theta} \sin(\alpha+\theta)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{Y}} = \frac{3}{2} M\ddot{Y} + m\ddot{Y} - m l \dot{\theta}^2 \cos(\alpha+\theta)$$

so,

$$(M+m) g \sin\alpha = \left(\frac{3}{2}M + m\right)\ddot{Y} - m l \dot{\theta}^2 \cos(\alpha+\theta)$$

$$\boxed{g \sin\alpha = \left(\frac{\frac{3}{2}M+m}{M+m}\right)\ddot{Y} - \left(\frac{m}{M+m}\right) l \dot{\theta}^2 \cos(\alpha+\theta)}$$

$$\frac{\partial L}{\partial \theta} = -m\dot{Y} l \dot{\theta} \cos(\alpha+\theta) - mgl \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta} - m\dot{Y} l \sin(\alpha+\theta)$$

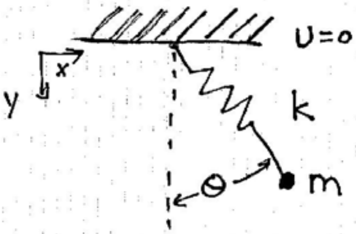
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m l^2 \ddot{\theta} - m\ddot{Y} l \sin(\alpha+\theta) - m\dot{Y} l \dot{\theta} \cos(\alpha+\theta)$$

so,

$$-m\dot{Y} l \dot{\theta} \cos(\alpha+\theta) - mgl \sin\theta = m l^2 \ddot{\theta} - m\ddot{Y} l \sin(\alpha+\theta) - m\dot{Y} l \dot{\theta} \cos(\alpha+\theta)$$

$$\boxed{-g \sin\theta = l \ddot{\theta} - \ddot{Y} \sin(\alpha+\theta)}$$

7-15



U=0

b = UNSTRETCHED SPRING LENGTH

ℓ = SPRING LENGTH.

$$x = \ell \sin\theta \Rightarrow \dot{x} = \dot{\ell}\sin\theta + \ell\dot{\theta}\cos\theta$$

$$y = \ell\cos\theta \Rightarrow \dot{y} = \dot{\ell}\cos\theta - \ell\dot{\theta}\sin\theta$$

$$\dot{x}^2 = \dot{\ell}^2\sin^2\theta + \ell^2\dot{\theta}^2\cos^2\theta + 2\dot{\ell}\ell\theta\sin\theta\cos\theta$$

$$\dot{y}^2 = \dot{\ell}^2\cos^2\theta + \ell^2\dot{\theta}^2\sin^2\theta - 2\dot{\ell}\ell\dot{\theta}\sin\theta\cos\theta$$

$$T = \frac{1}{2}m(\dot{\ell}^2 + \ell^2\dot{\theta}^2)$$

$$U = \frac{1}{2}k(\ell-b)^2 - mg\ell\cos\theta$$

$$L = T - U = \frac{1}{2}m(\dot{\ell}^2 + \ell^2\dot{\theta}^2) - \frac{1}{2}k(\ell-b)^2 + mg\ell\cos\theta$$

$$\frac{\partial L}{\partial \ell} = m\ell\dot{\theta}^2 - k(\ell-b) + mg\cos\theta$$

$$\frac{\partial L}{\partial \dot{\ell}} = m\dot{\ell}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = m\ddot{\ell}$$

$$m\ell\dot{\theta}^2 - k(\ell-b) + mg\cos\theta = m\ddot{\ell}$$

$$\boxed{\ddot{\ell} - \ell\dot{\theta}^2 + k(\ell-b) - g\cos\theta = 0}$$

7.15 CONT

$$\frac{\partial L}{\partial \theta} = -mg\ell \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\ell^2\dot{\theta}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = 2m\ell\dot{\ell}\dot{\theta} + m\ell^2\ddot{\theta}$$

So,

$$-mg\ell \sin\theta = \cancel{2m\dot{\ell}\ell}\ 2m\ell\dot{\ell}\dot{\theta} + m\ell^2\ddot{\theta}$$

$$\boxed{\ell\ddot{\theta} + 2\dot{\ell}\dot{\theta} + g\sin\theta = 0}$$