7.22

$L = T - U$

$\boxed{L = m\dot{x}^2/2 + (k/x)\, e^{-t/\tau}}$ w/ $F = -\frac{d}{dx}U$

$\boxed{H = E = T + U \neq \text{CONST}, \text{ SINCE } U = U(x,t)}$

ALSO THE CASE $\partial E/\partial t \neq 0$

$H = \frac{m}{2}\dot{x}^2 - \frac{k}{x}\, e^{-t/\tau}$, FROM ABOVE.

FROM EXPLICIT CONSTRUCTION

$H = P_x \dot{x} - L$

w/ $P_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$

$H = (m\dot{x})\dot{x} - \left(\frac{m}{2}\dot{x}^2 + \frac{k}{x}\, e^{-t/\tau}\right)$

$\boxed{H = \frac{m}{2}\dot{x}^2 - \frac{k}{x}\, e^{-t/\tau}}$

7.23

$$T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$U = U(x, y, z) \qquad \text{w/} \quad \underset{\sim}{F} = -\nabla U$$

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - U(x, y, z) \qquad (1)$$

$$H = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + U$$

$H = E$ b/c F is CONSERVATIVE

CANONICAL momenta

$$p_x = \frac{\partial L}{\partial \dot{x}} = m\dot{x}$$

$$p_y = m\dot{y}$$

$$p_z = m\dot{z}$$

REWRITE H:

$$\boxed{H = \frac{p_x^2}{2m} + p_y^2/2m + p_z^2/2m + U} \qquad (2)$$

Ham's EOM: $\dot{q}_j = \partial H / \partial p_j$

$$(3) \qquad \left.\begin{aligned} \Rightarrow \dot{x} &= p_x/m \\ \dot{y} &= p_y/m \\ \dot{z} &= p_z/m \end{aligned}\right\} \text{ALREADY KNOWN}$$

7-25

NOTE THAT $H = E = T + U$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = k\theta$$

$$\dot{x} = -r\dot{\theta}\sin\theta \qquad \therefore\ \dot{x}^2 = r^2\dot{\theta}^2\sin^2\theta$$

$$\dot{y} = r\dot{\theta}\cos\theta \qquad \therefore\ \dot{y}^2 = r^2\dot{\theta}^2\cos^2\theta$$

$$\dot{z} = k\dot{\theta} \qquad \dot{z}^2 = k^2\dot{\theta}^2$$

$$T = \frac{1}{2}m(r^2\dot{\theta}^2 + k^2\dot{\theta}^2)$$

$$U = mgz = mgk\theta$$

$$H = T + U = \frac{1}{2}m(r^2+k^2)\dot{\theta}^2 + mgk\theta$$

$$L = T - U = \frac{1}{2}m(r^2+k^2)\dot{\theta}^2 - mgk\theta$$

$$\frac{\partial L}{\partial\dot{\theta}} = P_\theta = m(r^2+k^2)\dot{\theta}$$

so,

$$\boxed{H = \frac{1}{2}\frac{P_\theta^2}{m(r^2+k^2)} + mgk\theta}$$

7.23 CON'T.

$$-\dot{p}_j = \partial H / \partial q_j$$

$$\Rightarrow \quad -\dot{p}_x = \partial H / \partial x = \partial U / \partial x$$

$$\dot{p}_x = -\partial U / \partial x = F_x$$ (CONSERVATIVE FORCE

$$\boxed{m\ddot{x} = F_x} \quad \text{FROM (3)} \qquad (4)$$

BY ANALOGY,

$$\boxed{\begin{aligned} m\ddot{y} &= F_y \\ m\ddot{z} &= F_z \end{aligned}} \qquad (5)$$

EQS (4) & (5) ARE NEWTON'S EOM.

7-25

NOTE THAT $H = E = T + U$

$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

$x = r\cos\theta$
$y = r\sin\theta$

$z = k\theta$

$\dot{x} = -r\dot{\theta}\sin\theta$ $\quad$ $\dot{x}^2 = r^2\dot{\theta}^2\sin^2\theta$

$\dot{y} = r\dot{\theta}\cos\theta$ $\quad$ $\dot{y}^2 = r^2\dot{\theta}^2\cos^2\theta$

$\dot{z} = k\dot{\theta}$ $\quad$ $\dot{z}^2 = k^2\dot{\theta}^2$

$T = \frac{1}{2} m (r^2\dot{\theta}^2 + k^2\dot{\theta}^2)$

$U = mgz = mgk\theta$

$H = T + U = \frac{1}{2} m (r^2 + k^2)\dot{\theta}^2 + mgk\theta$

$L = T - U = \frac{1}{2} m (r^2 + k^2)\dot{\theta}^2 - mgk\theta$

$\frac{\partial L}{\partial \dot{\theta}} = P_\theta = m(r^2 + k^2)\dot{\theta}$

SO,

$$\boxed{H = \frac{1}{2}\frac{P_\theta^2}{m(r^2+k^2)} + mgk\theta}$$

7-25 CONT

NOW,

$$\dot{\theta} = \frac{\partial H}{\partial P_\theta} = \frac{P_\theta}{m(r^2+k^2)}$$

$$\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = -mgk$$

HENCE

$$\ddot{\theta} = \frac{\dot{P}_\theta}{m(r^2+k^2)} = \frac{-gk}{(r^2+k^2)}$$

BUT $\ddot{z} = k\ddot{\theta}$

$$\Rightarrow \quad \ddot{z} = -\frac{gk^2}{(r^2+k^2)}$$

INTEGRATING

$$\boxed{z = -\frac{\frac{1}{2}gk^2t^2}{(r^2+k^2)}} \qquad z=0 \; @ \; t=0$$