M10.6.

SEVERAL WAYS TO THINK ABOUT IT. MAYBE THE EASIEST IS TO RECOGNIZE THAT WATER SURFACE IS $\perp$ TO DIRECTION OF $\underset{\sim}{g}_{EFF}$, THE EFFECTIVE acceleration DUE TO GRAVITY. THINK OF THE SURFACE OF A LAKE ON A CALM DAY. IT IS FLAT — PERPENDICULAR TO THE LOCAL DIRECTION OF $\underset{\sim}{g}$.

IN FRAME ROTATING w/ BUCKET

$$\underset{\sim}{F}_{EFF} = \underset{\sim}{S} + m\underset{\sim}{g} - m\,\dot{\underset{\sim}{\omega}} \times \underset{\sim}{r} - 2m\,\underset{\sim}{\omega} \times \dot{\underset{\sim}{r}} - m\,\underset{\sim}{\omega} \times (\underset{\sim}{\omega}$$

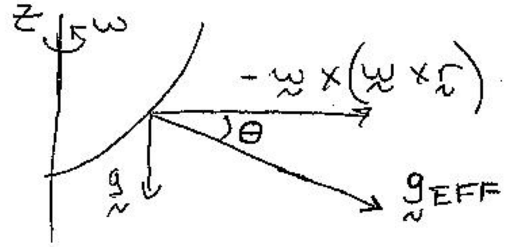
w/ $\underset{\sim}{S}$ = REAL FORCES

m = "CHUNK" OF WATER

$\underset{\sim}{g}$ = GRAVITATIONAL ACCELERATION

IN ROT FRAME

$$\underset{\sim}{g}_{EFF} = \underset{\sim}{g} - \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$$



M 10.6 CONT

$\tan\theta = \dfrac{g}{\omega^2 r}$ = SLOPE OF LINE $\perp$ TO WATER SURFACE

$dz/dr = -\tan\theta$ ($-\underset{\sim}{g}_{EFF}$ POINTS OUT OF H_2O)

SO, WATER SURFACE $\perp$ TO HIS DIRECTION HAS $dz/dr = 1/\tan\theta$.

RECALL THAT 2 LINES $\perp$ TO EACH OTHER HAVE SLOPES $m_1 m_2 = -1$.

FOR WATER SURFACE

$$dz/dr = \frac{\omega^2}{g} r$$

$$\boxed{z = \frac{\omega^2}{g} r^2 + C_0}$$

$\therefore$ SURFACE IS PARABALOID

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M 10.7

$$\underset{\sim}{F}_{EFF} = m \underset{\sim}{g}_{EFF} = \frac{G M_E m \hat{r}}{r^2} - m \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$$

DIFFERENCE BETWEEN POLAR AND EQUATORIAL $\underset{\sim}{g}$ IS DUE TO $\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$

$$|\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})| = \omega^2 R_E$$

$$\boxed{\Delta g = 34 \ mm/s^2}$$

WHERE $\omega = 7.3 \times 10^{-5}$ RAD/s

$R_E = 6.38 \times 10^6$ M (EQUATORIAL RADIUS

DIFFERENCE BETWEEN $34 \ mm/s^2$ & $52 \ mm/s^2$ IS DUE TO NON-SPHERICITY OF EARTH. EARTH BULGES AROUND EQUATOR. THIS "BELT" OR BULGE PULLS MORE AT POLES THAN AT EQUATOR.

10-9

PROBLEM RESEMBLES EX 10.3, P393.

IN ROTATING REF FRAME:

$$\underset{\sim}{a_r} = \underset{\sim}{g} - 2\, \underset{\sim}{\omega} \times \underset{\sim}{v_r}$$

"EFFECTIVE" GRAVITATIONAL ACCELERATION INCLUDES $\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$ TERM.

SEE FIG 10-9, P. 394.

NOW,

$$\underset{\sim}{\omega} \times \underset{\sim}{v_r} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\omega\cos\lambda & 0 & \omega\sin\lambda \\ 0 & V_0\cos\alpha & V_0\sin\alpha - gt \end{vmatrix}$$

$\hat{x}$: $-\omega\sin\lambda\, V_0\cos\alpha$

$\hat{y}$: $+\omega\cos\lambda\,(V_0\sin\alpha - gt)$

$\hat{z}$: $-\omega\cos\lambda\, V_0\cos\alpha$

SO,

$$(a_r)_x = +2\omega V_0 \sin\lambda\cos\alpha \quad \text{(MINUS SIGN FROM (}$$

$$(a_r)_y = -2\omega V_0\cos\lambda\sin\alpha + g\omega\cos\lambda\, t$$

$$(a_r)_z = +2\omega V_0\cos\lambda\cos\alpha - g$$

10-9 CONT.

DEFLECTION IN X-Y PLANE, PARALLEL TO X-AXIS.

$$(a_r)_x = \ddot{x} = +\, 2\omega V_0 \sin\lambda \cos\alpha$$

$$\Rightarrow x = +\, \omega V_0 \sin\lambda \cos\alpha\, t^2; \qquad (2)$$

WHERE $x(t=0) = \dot{x}(t=0) = 0$.

FOR FLIGHT TIME, USE $T = 2V_0 \sin\alpha / g$ (SEE EQ 2.37, p 64).

SUBSTITUTING THIS VALUE FOR t,

$$\boxed{x = +\, \frac{4V_0^3}{g}\, \omega \sin\lambda \sin^2\alpha \cos\alpha}$$

F14.

ASSUME 3 LITERS/DAY.

AVERAGE LIFETIME $\simeq$ 80 YRS

TTL VOLUME $\simeq 3 \times 365 \times 80$ LITERS

$\boxed{\text{TTL VOL} \simeq \cancel{9 \times 10}\ 10^5 \text{ LITERS}}$

ANSWER PROBABLY ACCURATE TO WITHIN A FACTOR OF 2 OR 50