BEVINGTON

6.2

$$\sigma_a^2 = \frac{1}{\Delta} \Sigma x_i^2 / \sigma_i^2$$

$$\sigma_b^2 = \frac{1}{\Delta} \Sigma \frac{1}{\sigma_i^2}$$

w/ $\Delta = \Sigma \frac{1}{\sigma_i^2} \Sigma \frac{x_i^2}{\sigma_i^2} - \left(\Sigma \frac{x_i}{\sigma_i^2} \right)^2$

LET $\sigma = \sigma_i$

$$\Rightarrow \Delta = \frac{1}{\sigma^2} N \cdot \frac{1}{\sigma^2} \Sigma x_i^2 - \frac{1}{\sigma^4} (\Sigma x_i)^2$$

$$= \frac{1}{\sigma^4} \left[N \Sigma x_i^2 - (\Sigma x_i)^2 \right]$$

$$= \Delta' / \sigma^4$$

HEARLE'S $\sigma_a = \frac{\sigma^4}{\Delta'} \Sigma \frac{x_i^2}{\sigma_0^2} = \frac{\sigma^2}{\Delta'} \Sigma x_i^2$

$$\boxed{\sigma_a = \frac{\sigma^2}{\Delta'} \Sigma x_i^2}$$

w/ $\Delta' = N \Sigma x_i^2 - (\Sigma x_i$

$$\sigma_b = \frac{\sigma^4}{\Delta'} \cdot \frac{1}{\sigma^2} N$$

$$\boxed{\sigma_b = N \frac{\sigma^2}{\Delta'}}$$

6.5 LEAST SQUARES FITTING

$F = k\Delta l$

$mg = k(l - l_0)$

$l = (g/k)m + l_0$

m = INDEPENDENT VARIABLE

LET $b = (g/k)$

$a = l_0$

$y = l$

$y = a + bx$

USE A CALCULATOR

$a = 3.69$ cm $\Rightarrow$ $\boxed{l_0 = 3.7 \text{ cm}}$

$b = 6.07 \times 10^{-3}$ cm/kg $\leftarrow$ UNITS

$b = g/k \Rightarrow k = g/b$, $g = 9.8$ m/s^2

$\boxed{k = 1.6 \times 10^{5} \text{ kg/s}^2}$

6.5 Now, $\sigma_A^2 = \frac{\sigma^2}{\Delta'} \sum X_i^2$

w/ $\sigma = 0.2$ cm

$\Delta' = N \sum X_i^2 - (\sum X_i)^2$

$\Delta' = 3.36 \times 10^6$ cm^2

w/ $\sum X_i^2 = 2.84 \times 10^6$ cm^2

$\sum X_i = 4.4 \times 10^3$ cm

$\Rightarrow \sigma_A^2 = 3.38 \times 10^{-2}$ cm^2

$\therefore \sigma_A = 1.84 \times 10^{-1}$ cm

$\sigma_b^2 = N \sigma^2 / \Delta'$

$= 9.52 \times 10^{-8}$ cm^2/kg^2

$\sigma_b = 3.1 \times 10^{-4}$ cm/kg

Now, $k = g/b \Rightarrow \sigma_k = (g/b^2)\sigma_b$

w/ $g = 980$ cm/s^2 & $b = 6.07 \times 10^{-3}$ cm/kg

$\sigma_k = 8.24 \times 10^3$ kg/s^2

6.5 $\chi^2 = \sum \left(\frac{y_i - a - b x_i}{\sigma_i} \right)^2$

Y_{th} (cm)	Y_{EXP} (cm)	$\left(\frac{Y_{EXP} - Y_{th}}{\sigma} \right)^2$
4.9	5.1	1
5.5	5.5	0
6.1	5.9	1
6.7	6.8	0.25
7.3	7.4	0.25
7.9	7.5	4
8.5	8.6	.25
9.2	9.4	1

$\Rightarrow$ $\boxed{\chi^2 = 7.75 \quad \text{w/} \quad \nu = 6}$

$\chi^2_{.9} = 1.3$

$\Rightarrow$ $\boxed{P \simeq 26\%}$

VIA TABLE C.4 AND LINEAR INTERPOLATION.