BEVINGTON PHYSICS 4211 T. COAN

3.1., P50.

(a) $X = \frac{1}{2(u+v)}$ $\frac{\partial X}{\partial u} = -\frac{1}{2}(u+v)^{-2}$ $\frac{\partial X}{\partial v} = -\frac{1}{2}(u+v)^{-2}$

$$\Rightarrow \boxed{\sigma_x^2 = \frac{1}{4}\frac{1}{(u+v)^4}\sigma_u^2 + \frac{1}{4}\frac{1}{(u+v)^4}\sigma_v^2 + \frac{\sigma_{uv}}{2}\frac{1}{(u+v)^4}}$$

OK TO ASSUME $\sigma_{uv} = 0$.

(b) $X = \frac{1}{2(u-v)}$ $\frac{\partial X}{\partial u} = -\frac{1}{2}(u-v)^{-2}$ $\frac{\partial X}{\partial v} = +\frac{1}{2}(u-v)^{-2}$

$$\boxed{\sigma_x^2 = \frac{1}{4}(u-v)^{-4}\sigma_u^2 + \frac{1}{4}(u-v)^{-4}\sigma_v^2 - \frac{\sigma_{uv}}{2}(u-v)^{-4}}$$

OK TO ASSUME $\sigma_{uv} = 0$.

(c) $X = 1/u^2 \Rightarrow \boxed{\sigma_x^2 = 4u^{-6}\sigma_u^2}$

d) $X = uv^2$ $\frac{\partial X}{\partial u} = v^2$ $\frac{\partial X}{\partial v} = 2uv$

$$\boxed{\sigma_x^2 = v^4\sigma_u^2 + 4u^2v^2\sigma_v^2 + 4uv^3\sigma_{uv}^2}$$

e) $X = u^2 + v^2$ $\frac{\partial X}{\partial u} = 2u$ $\frac{\partial X}{\partial v} = 2v$

$$\boxed{\sigma_x^2 = 4u^2\sigma_u^2 + 4v^2\sigma_v^2 + 8uv\,\sigma_{uv}^2}$$

OK TO ASSUME $\sigma_{uv}^2 = 0$.

T. COAN

$\boxed{3.2, p51.}$

$$A = \pi d^2/4$$

$$\sigma_A^2 = \left(\frac{\pi d}{2}\right)^2 \sigma_d^2$$

$$\Rightarrow \frac{\sigma_A}{A} = 2\ \sigma_d/d$$

For $\sigma_d/d = 10^{-2}$, $\boxed{\sigma_A/A = 0.02}$

NO, MEASURING THE RADIUS WOULD <u>NOT</u> REDUCE σ_A/A IF $\sigma_r/r = \sigma_d/d = 10^{-2}$.

$\boxed{3.4, p51.}$

$$N = N_0\, e^{-t/\tau}$$

GOAL: COMPUTE σ_N^2/N^2

NOTE: $\dfrac{\partial N}{\partial N_0} = e^{-t/\tau}$

$$\partial N/\partial\tau = N_0 \frac{t}{\tau^2} e^{-t/\tau}$$

$$\Rightarrow \sigma_N^2 = e^{-2t/\tau}\sigma_{N_0}^2 + N_0^2\left(\frac{t}{\tau^2}\right)^2 e^{-2t/\tau}\sigma_\tau^2$$

or

$$\sigma_N^2/N^2 = \underset{①}{\frac{\sigma_{N_0}^2}{N_0^2}} + \underset{②}{\left(\frac{t}{\tau}\right)^2\frac{\sigma_\tau^2}{\tau^2}}$$

TERMS ① & ② CONTRIBUTE EQUALLY TO $\dfrac{\sigma_N^2}{N^2}$ WHEN $t = \tau$ IF $\left|\dfrac{\sigma_{N_0}}{N_0}\right| = \left|\sigma_\tau/\tau\right|$.

T. COAN

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3.6, p51.

LET $z = \Delta\nu$, THEN

$$z = \frac{v\nu}{u-v}$$

$$\frac{\partial z}{\partial \nu} = \frac{v}{u-v} \; ; \quad \frac{\partial z}{\partial v} = \frac{u\nu}{(u-v)^2} \; ; \quad \frac{\partial z}{\partial u} = \frac{-v\nu}{(u-v)^2}$$

$$\Rightarrow \sigma_z^2 = \frac{v^2}{(u-v)^2}\sigma_\nu^2 + \frac{(v\nu)^2}{(u-v)^4}\sigma_u^2 + \frac{(u\nu)^2}{(u-v)^4}\sigma_v^2$$

NOW, $u - v \simeq u$ (FROM THE DATA), SO

$$\sigma_z^2 \simeq \left(\frac{v}{u}\right)^2 \nu^2 \left(\frac{\sigma_\nu^2}{\nu^2}\right) + \left(\frac{v}{u}\right)^2 \nu^2 \left(\frac{\sigma_u^2}{u^2}\right) + \left(\frac{v}{u}\right)^2 \nu^2 \left(\frac{\sigma_v^2}{v^2}\right)$$

$$\simeq \underset{1}{g\left(\frac{\sigma_\nu}{\nu}\right)^2} + \underset{2}{g\left(\frac{\sigma_u}{u}\right)^2} + \underset{3}{g\left(\frac{\sigma_v}{v}\right)^2} \; ; \quad g = \left(\frac{v\nu}{u}\right)^2$$

THE TERM(S) w/ THE LARGEST "FRACTIONAL VARIANCE" ONLY SHOULD BE RETAINED.

FROM THE DATA: $\frac{\sigma_\nu}{\nu} = \frac{1}{1000} = 10^{-3}$

$\sigma_u/u = 5/332 = 1.51 \times 10^{-2}$

$\sigma_v/v = \frac{0.003}{0.123} = 2.44 \times 10^{-2}$

HENCE, IGNORE 1.

$\Rightarrow \boxed{\sigma_z \simeq 1.06 \times 10^{-2}\ Hz}$, where I HAVE USED

$\bar{\nu} = 1000\ Hz$

$\bar{u} = 332\ m/s$ & $g = 1.37 \times 10^{-1}\ Hz^2$

$\bar{v} = 0.123\ m/s$

BEVINGTON 3.10, P 51.

FOR "NUMERICAL INTEGRATION," SEE APPENDIX C.2, P 253.

THE NAME OF THE GAME IS TO COMPUTE $A_G(z)$ BECAUSE $P_G(z > z_0) = 1 - A_G(z_0)$; $z = \frac{x-\mu}{\sigma}$

(a) $P_G(z>1) = 1 - A_G(1)$

$= 1 - 0.6827$

$\boxed{P(z>1) = 0.317}$

(b) $P_G(z>2) = 1 - A_G(2)$

$= 1 - 0.9545$

$\boxed{P_G(z>2) = 0.0455}$

(c) $P_G(z>3) = 1 - A_G(3)$

$= 1 - 0.9973$

$\boxed{P_G(z>3) = 0.0027}$