PHYS 5382
Fall 2016
TE Coan
Due: 21 Oct '16 6:00 pm

Homework 4

0. Box your **entire** answer for each problem or lose points.

1. Show that the operator $\hat{C}$ defined through $[\hat{A}, \hat{B}] = i\hat{C}$ is hermitian if the operators $\hat{A}$ and $\hat{B}$ are. The result of problem 2.9 may be useful.

2. In lecture I claimed for the operators $\hat{A}, \hat{B}$ and $\hat{C}$, that $[\hat{A}, \hat{B}\hat{C}] = \hat{B}[\hat{A}, \hat{C}] + [\hat{A}, \hat{B}]\hat{C}$. Verify this explicitly. Box your final answer.

3. A spin-1 particle is in the state

$$|\Psi\rangle \underset{\mathrm{S_z basis}}{\longrightarrow} \frac{1}{\sqrt{14}} \begin{pmatrix} i \\ 2 \\ 3i \end{pmatrix}.$$

What are the probabilities that a measurement of S_z will yield the values $\hbar, 0, -\hbar$ for this state? What is $\langle S_z \rangle$?

(b.) What is $\langle S_x \rangle$? Note that the matrix operator for S_x and J_x we saw in lecture are the same in this case since they both refer to spin.

(c.) POSTPONED What is the probability that a measurement of S_x will yield the value of $\hbar$ for this state?

4. A bit of drill. Use the matrix representation of the spin-$\frac{1}{2}$ angular mopmentum operators $\hat{S}_x, \hat{S}_y$ and $\hat{S}_z$ in the S_z basis to verify explicitly through matrix multiplication that

$$[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z.$$

5. Determine the matrix representation of the spin-$\frac{1}{2}$ angular momentum operators $\hat{S}_x, \hat{S}_y$ and $\hat{S}_z$ using the eigenstates of $\hat{S}_y$ as a basis. Note that the commutation relations among the various $\hat{S}$ operators are independent of the basis you are working in. This can sometimes save you time.