TEC.

14.10

FIND EIGENVALUES OF L_x:

$$L_x = \frac{L_+ + L_-}{2} \qquad (1)$$

$$= \hbar \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad (2)$$

$$L_x \begin{pmatrix} a \\ b \\ c \end{pmatrix} = m\hbar \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (3)$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \sqrt{2}\, m \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (4)$$

L_x behaves similar to L_z in the sense that possible values of m_x FOR $\ell = 1$ ARE: $m = \pm 1, 0$

$m = 1$: EQ (4) YIELDS

$$\left.\begin{aligned} b &= \sqrt{2}\, a \\ a + c &= \sqrt{2}\, b \\ b &= \sqrt{2}\, c \end{aligned}\right\} \Rightarrow a = c, \quad b = \sqrt{2}\, a$$

CHOOSE $a = 1$,

$\Rightarrow \quad c = 1, \; b = \sqrt{2}$

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$$|l=1\ m_x=1\rangle = C_0 \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$$

$$\langle l=1\ m_x=1 | l=1\ m_x=1\rangle = 1$$

$$\Rightarrow C_0 = 1/2$$

So, $$\boxed{|l=1\ m_x=1\rangle = \begin{pmatrix} 1/2 \\ 1/\sqrt{2} \\ 1/2 \end{pmatrix}}$$

$m = 0$

$$b = 0$$

$$a + c = 0$$

CHOOSE $a = 1$, $c = -1$, $b = 0$

w/ NORMALIZATION

$$|l=1\ m_x=0\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

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FINALLY,

$m = -1$

FROM EQ (4)

$$b = -\sqrt{2}\, a$$
$$a + c = -\sqrt{2}\, b$$
$$b = -\sqrt{2}\, c$$

SO, w/ NORMALIZATION

$$|\ell=1 \; m_x=-1\rangle = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{pmatrix}$$

YOU CAN ALSO HAVE

$$|\ell=1 \; m_x=-1\rangle = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{2} \end{pmatrix}$$

NOW WE COMPUTE

$$P(m_x=0) = |\langle u | \ell=1 \; m_x=0\rangle|^2$$

$$= \left[\frac{1}{\sqrt{26}}\begin{pmatrix}1 & 4 & -3\end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}\right]^2 = \left(\frac{\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}}{\sqrt{26}}\right)^2 = \frac{16}{52} = \frac{4}{13}$$

$$\boxed{P(m_x=0) = 4/13}$$

14.02

$$S_x + S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \frac{\hbar}{2}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}$$

FIND E.V.'s & E.F.'s.

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{2}{\hbar}\lambda \begin{pmatrix} a \\ b \end{pmatrix} \quad (2)$$

$$\Rightarrow \left\| \begin{matrix} -\lambda' & 1-i \\ 1+i & -\lambda' \end{matrix} \right\| = 0 \qquad w/\ \lambda' = \frac{2}{\hbar}\lambda \quad (3)$$

$$\lambda'^2 - (1+1) = 0$$

$$\lambda' = \pm\sqrt{2}$$

$$\therefore \boxed{\lambda = \pm\frac{\hbar}{\sqrt{2}}}$$

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FOR $\lambda = -\hbar/\sqrt{2}$, CALCULATION IS VERY SIMILAR:

$$b(1-i) = -\sqrt{2}\,a$$

$$a(1+i) = -\sqrt{2}\,b$$

$$\Rightarrow \left.\begin{aligned} e^{-i\pi/4}\,b &= -a \\ e^{+i\pi/4}\,a &= -b \end{aligned}\right\} \Rightarrow |a| = |b| \text{ AS BEF}$$

SET $a = 1$, SO $b = -\dfrac{(1+i)}{\sqrt{2}}$

INCLUDING NORMALIZATION, THE EIGENFUNCT FOR $\lambda = -\hbar/\sqrt{2}$ IS

$$\boxed{\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{(1+i)}{2} \end{pmatrix}}$$

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NOW FIND EIGENFUNCTIONS

FOR $\lambda = +\hbar/\sqrt{2}$

$$(S_x + S_y)\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \quad b(1-i) = \sqrt{2}\, a$$
$$a(1+i) = \sqrt{2}\, b$$

OR

$$\left.\begin{aligned} e^{-i\pi/4}\, b &= a \\ e^{+i\pi/4}\, a &= b \end{aligned}\right\} \Rightarrow |a| = |b|$$

SET ARBITRARILY $a = 1$, THEN $b = e^{+i\pi/4}$
$= \frac{1+i}{\sqrt{2}}$

SO, E.F. FOR $\lambda = +\hbar/\sqrt{2}$:

$$\boxed{\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1+i}{2} \end{pmatrix}}$$

INCLUDES NORMALIZATION FACTOR

i.e., $(a\ b)\begin{pmatrix} a \\ b \end{pmatrix} = 1$

14.12

Prob (measuring $S_z = \hbar/2$) $= |\langle s=\tfrac{1}{2}\, s_z=\tfrac{1}{2} \,|\, s=\tfrac{1}{2}, S_x+S_y =$

$$= \left| \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1+i}{2} \end{pmatrix} \right|^2$$

$$= \left| \frac{1}{\sqrt{2}} \right|^2$$

$$\boxed{\text{Prob} = 1/2}$$