GR 4.36

(a) ADD 3 $1/2$ - SPINS

POSSIBLE VALUES FOR 2 $1/2$-SPINS: 0, 1
NOW ADD FINAL $1/2$.

$$\Rightarrow S_T = \tfrac{1}{2} \quad (0 + 1/2)$$
$$\tfrac{3}{2}, \tfrac{1}{2} \quad (1 + 1/2)$$

SO, $\boxed{\text{BARYON SPIN} = 1/2,\ 3/2}$

(b) FOR MESONS, ADD 2 SPIN - $1/2$'s.

$$\boxed{S_{TOT} = 0, 1}$$

GA 15.9

FOR $S_z = 0$, $S_{1z} + S_{2z} = 0$

SO, $S_{1z} S_{2z} = -1/4$

$$V(r) = V_1(r) + V_2\left[12\left(-1/4\right) - 2\left[2 - \frac{3}{4} - \frac{3}{4}\right]\right]$$

$$+ 2V_3\left[2 - 3/4 - 3/4\right]$$

$$V(r) = V_1(r) + V_2\left[-3-1\right] + V_3$$

$$\boxed{V(r) = V_1(r) - 4V_2(r) + V_3(r)}$$ $|10\rangle$ STATE

#3 FROM C-G TABLES

$$\boxed{\left|\tfrac{5}{2}\,\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{6}{35}}\,|22\rangle\left|\tfrac{3}{2}\,-\tfrac{3}{2}\right\rangle + \sqrt{\tfrac{5}{14}}\,|21\rangle\left|\tfrac{3}{2}\,-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{3}{35}}\,|20\rangle\left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{27}{70}}\,|2\,-1\rangle\left|\tfrac{3}{2}\,\tfrac{3}{2}\right\rangle}$$

#4 $$|22\rangle|2\,-1\rangle = \sqrt{\tfrac{1}{14}}\,|41\rangle + \sqrt{\tfrac{3}{10}}\,|31\rangle + \sqrt{\tfrac{3}{7}}\,|21\rangle + \sqrt{\tfrac{1}{5}}\,|11\rangle$$

GA 15.9

FOR $S=0$:

$$V(r) = V_1 + V_2\left[-3 - 2\left[S(S+1) - S_1(S_1+1) - S_2(S_2+1)\right]\right]$$

$$+ 2V_3\left[S(S+1) - S_1(S_1+1) - S_2(S_2+1)\right]$$

$$= V_1 + V_3\left[-3 - 2\left(0 - \frac{3}{4} - \frac{3}{4}\right)\right] + 2V_3\left[0 - \frac{3}{4} - \frac{3}{4}\right]$$

$$\boxed{V(r) = V_1 - 3V_3 \qquad S=0 \text{ STATE}}$$

WHERE WE HAVE USED $S^2|S m_s\rangle = S(S+1)|S m_s\rangle$

AND FOR $S_z = 0$, $S_1 = \pm 1/2$, $S_2 = \mp S_2$

SINCE $S_z = S_{1z} + S_{2z}$

FOR TRIPLET, $S=1$, $S_z = \pm 1, 0$

FOR $S=1$, $S_z \pm 1$ STATES, $S_{1z} S_{2z} = +1/4$

$$V(r) = V_1(r) + V_2\left[12\left(\frac{1}{4}\right) - 2\left[2 - \frac{3}{4} - \frac{3}{4}\right]\right]$$

$$+ 2V_3\left[2 - \frac{3}{4} - \frac{3}{4}\right]$$

$$= V_1(r) + V_2\left[3 - 2\left(\frac{1}{2}\right)\right] + 2V_3\left[1/2\right]$$

$$\boxed{V(r) = V_1(r) + 2V_2(r) + V_3(r) \qquad S=1, S_z = \pm 1 \text{ STATES}}$$

GA 15.9

(a) $S_T = 0$ (SINGLET STATE)

$\underset{\sim}{S} = \frac{\hbar}{2} \underset{\sim}{\sigma}$ (WE WILL IGNORE $\hbar$)

LET $\underset{\sim}{r}$ be aligned w/ Z-AXIS
HINT ALSO GIVEN IN GA 15.8

$\Rightarrow \underset{\sim}{S}_1 \cdot \underset{\sim}{r} = S_{1z} z$

$r^2 = z^2$

HENCE

$V(r) = V_1(r) + V_2(r)\left[12\, S_{1z} S_{2z} - 4\, \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2\right] + V_3\, 4\underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2$

Now, $\underset{\sim}{S} = \underset{\sim}{S}_1 + \underset{\sim}{S}_2$

$S^2 = S_1^2 + S_2^2 + 2\underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2$

$\Rightarrow \underset{\sim}{S}_1 \cdot \underset{\sim}{S}_2 = \dfrac{S^2 - S_1^2 - S_2^2}{2}$