5383/5Ø3 T.E.C.

GA 15.10

$$|S=0\rangle = \frac{1}{\sqrt{2}}\left\{ \underset{e1}{|\tfrac{1}{2}\,\tfrac{1}{2}\rangle} \underset{e2}{|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle} - \underset{e1}{|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle} \underset{e2}{|\tfrac{1}{2}\,\tfrac{1}{2}\rangle} \right\}$$

(a) $S_z(\text{TOTAL}) = 0 \Rightarrow S_{1z} = -S_{2z}$

$$\boxed{\therefore \ \text{Prob}\left(S_{2z} = +\tfrac{1}{2}\right) = 0}$$

(b) $\psi(\text{SINGLET}) = \frac{1}{\sqrt{2}}\left(\chi_+(1)\chi_-(2) - \chi_-(1)\chi_+(2)\right)$

w/ $\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$, $\chi_- = \begin{pmatrix}0\\1\end{pmatrix}$

WE SEEK EXPLICIT FORM OF $\left(S_{1y} = \frac{1}{2}, S_{2x} = -\frac{1}{2}\right)$ STATE

FOR $S_{1y} = 1/2$:

$$\frac{1}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = \frac{1}{2}\begin{pmatrix}a\\ b\end{pmatrix}$$

$$\Rightarrow \begin{aligned} -ib &= a \\ ia &= b \end{aligned}$$

SET $a = 1$ AND NORMALIZE.

$$|S_{1y} = 1/2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} = \frac{1}{\sqrt{2}}\left(\chi_+ + i\chi_-\right)$$

GH 15.10

$|S_x = -\tfrac{1}{2}\rangle$:

$$\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow b = -a$$

$$|S_x = -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\chi_+ - \chi_-)$$

So,

$$|S_{1y} = \tfrac{1}{2},\ S_{2x} = -\tfrac{1}{2}\rangle = \frac{1}{2}\left(\chi_+(1) + i\chi_-(1)\right)\left(\chi_+(2) - \chi_-(2)\right)$$

$$\text{PROB}\left(S_{1y} = \tfrac{1}{2},\ S_{2x} = -\tfrac{1}{2}\right) = \left|\langle S_{1y} = \tfrac{1}{2}\ S_{2x} = -\tfrac{1}{2}\,|\,S=0\rangle\right|^2$$

$$= \left[\frac{1}{2}\left(\chi_+^\dagger(1) - i\chi_-^\dagger(1)\right)\left(\chi_+^\dagger(2) - \chi_-^\dagger(2)\right)\left(\chi_+(1)\chi_-(2) - \chi_-(1)\chi_+(2)\right)\right]^2$$

$$= \frac{1}{8}\Big[\left(\chi_+^\dagger(1)\chi_+(1) - i\chi_-^\dagger(1)\chi_+(1)\right)\left(\chi_+^\dagger(2)\chi_-(2) - \chi_-^\dagger(2)\chi_-(2)\right)$$
$$- \left(\chi_+^\dagger(1)\chi_-(1) + i\chi_-^\dagger(1)\chi_-(1)\right)\left(\chi_+^\dagger(2)\chi_+(2) - \chi_-^\dagger(2)\chi_+(2)\right)\Big]^2$$

$$= \frac{1}{8}\left[(1-0)(0-1) - (0+i)(1)\right]^2$$

$$\boxed{\text{PROB} = 1/4}$$

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GA 15.10

(c) $P(\text{TRIPLET}) = 1 - P(\text{SINGLET})$

$$\psi(\text{SINGLET}) = \frac{1}{\sqrt{2}}\left(\chi_+(1)\chi_-(2) - \chi_-(1)\chi_+(2)\right)$$

$$P(\text{SINGLET}) = \Big|\langle\psi|\left(\cos\alpha_1\,\chi_+(1) + \sin\alpha_1\, e^{i\beta_1}\chi_-(1)\right)$$

$$\times\left(\chi_+(2)\cos\alpha_2 + \sin\alpha_2\, e^{i\beta_2}\chi_-(2)\right)\rangle\Big|^2$$

$$= \Big|\left(\cos\alpha_1\,\chi_+^\dagger(1)\chi_+(1) + \sin\alpha_1 e^{i\beta_1}\chi_+^\dagger(1)\chi_-(1)\right)$$

$$\times\left(\chi_-^\dagger(2)\cos\alpha_2\,\chi_+(2) + \chi_-^\dagger(2)\sin\alpha_2 e^{i\beta_2}\chi_-(2)\right)$$

$$-\left(\chi_-^\dagger(1)\cos\alpha_1\,\chi_+(1) + \chi_-^\dagger(1)\sin\alpha_1 e^{i\beta_1}\chi_-(1)\right)$$

$$\times\left(\chi_+^\dagger(2)\cos\alpha_2\,\chi_+(2) + \chi_+^\dagger(2)\sin\alpha_2 e^{i\beta_2}\chi_-(2)\right)\Big|^2$$

$$= \left|\cos\alpha_1\sin\alpha_2\, e^{i\beta_2} - \sin\alpha_1 e^{i\beta_1}\cos\alpha_2\right|^2$$

$$= \left|\cos\alpha_1\sin\alpha_2 - \sin\alpha_1\cos\alpha_2\, e^{i(\beta_1-\beta_2)}\right|^2$$

$$P(\text{SINGLET}) = \cos^2\alpha_1\sin^2\alpha_2 + \sin^2\alpha_1\cos^2\alpha_2$$

$$- 2\cos\alpha_1\sin\alpha_1\sin\alpha_2\cos\alpha_2\cos(\beta_1-\beta_2)$$

$P(\text{TRIPLET} = 1 - P(\text{SINGLET}$

6A16.2

$$H' = \frac{-3e^2}{2R^3}\left(R^2 - \frac{1}{3}r^2\right) \qquad r < R \tag{1}$$

$$= 0 \qquad r > R$$

STRAIGHTFORWARD CALC, APPROXIMATION IS HELPFUL.

$$E_{10}^{(1)} = \int_0^R dr\, r^2 \left(\frac{-3e^2}{2R}\right) 4\left(\frac{1}{a_0}\right)^3 e^{-2r/a_0}$$

$$+ \int_0^R dr\, r^2 \frac{e^2 r^2}{2R^3} 4\left(\frac{1}{a_0}\right)^3 e^{-2r/a_0}$$

$$= \frac{-6e^2}{R a_0^3} \underbrace{\int_0^R dr\, r^2 e^{-2r/a_0}}_{1}$$

$$+ \frac{2e^2}{R^3 a_0^3} \underbrace{\int_0^R dr\, r^4 e^{-2r/a_0}}_{2} \tag{2}$$

LET $r/a_0 = z$ $\{$ b/c $R/a_0 \ll 1$, $z \ll 1$

$$e^{-2r/a_0} = e^{-2z} \simeq 1 - 2z$$

$$\int 1 \simeq \frac{-6e^2}{R}\int_0^{R/a_0} dz\, z^2(1-2z)$$

$$\simeq \frac{-6e^2}{R}\left[\frac{z^3}{3} - \frac{1}{2}z^4\right]\Big|_0^{R/a_0}$$

©

GA 162

$$\int (2) = + \frac{2e^2 a_0^2}{R^3} \int_0^{R/a_0} dz\, z^4 e^{-2z}$$

$$\simeq \frac{2e^2 a_0^2}{R^3} \int_0^{R/a_0} dz\, z^4 (1-2z)$$

$$\simeq \frac{2e^2 a_0^2}{R^3} \left[\frac{z^5}{5} - \frac{z^6}{3} \right] \Bigg|_0^{R/a_0}$$

DOMINANT TERM $\to z^5$

$$\boxed{\begin{aligned} \Delta E_{1 0}^{(1)} &\simeq - \frac{2e^2 R^2}{a_0^3} && \text{c.g.s} \\ &\simeq - \frac{2e^2 R^2}{(4\pi\epsilon_0) a_0^3} && \text{m.k.s} \end{aligned}}$$

GA 16.2

USE APPROXIMATION TO PICK OUT LEADING TERM

$$\Delta E^{(1)}_{20} = \frac{-6e^2}{8Ra_0^3}\int_0^R dr\, r^2\left(1-\frac{r}{2a_0}\right)^2 e^{-r/a_0}$$
(1)

$$+\frac{2e^2}{8R^3a_0^3}\int_0^R dr\, r^4\left(1-\frac{r}{2a_0}\right)^2 e^{-r/a_0}$$
(2)

$$\int (1) \simeq \frac{-6e^2}{8Ra_0^3}\int dr\, r^2\left(1-\frac{r}{2a_0}\right)^2\left(1-r/a_0\right)$$

$$\simeq \frac{-6e^2}{8R}\int_0^{R/a_0} dz\, z^2\left(1-\frac{z}{2}\right)^2(1-z)$$

$$\simeq \frac{-6e^2}{8R}\left.\frac{z^3}{3}\right|_0^{R/a_0}$$

$$\boxed{\Delta E^{(1)}_{20} \simeq -\frac{e^2R^2}{4a_0^3} \qquad \text{c.g.s}}$$

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GA 16.2

FINALLY,

$$\Delta E_{21}^{(1)} \simeq \frac{-3e^2}{2R} * \frac{1}{3} \frac{1}{8a_0^3} \int_0^R dr\, r^2 \frac{r^2}{a_0^2} e^{-r/a_0}$$

$$\simeq \frac{-e^2}{16R} \int_0^{R/a_0} dz\, z^4 \, (1-z)$$

$$\simeq \frac{-e^2}{16R} \, z^5/5 \Big|_0^{R/a_0}$$

$$\boxed{\Delta E_{21}^{(1)} \simeq \frac{-e^2 R^4}{80\, a_0^5}}$$

(9)

GA 16.3

$$\Delta E_0^{(1)} = C_0 \langle 0 | A A^\dagger A A^\dagger + A^2 A^{\dagger 2} | 0 \rangle$$

now, $A^\dagger |0\rangle = |1\rangle$ & $A|1\rangle = |0\rangle$

$A^\dagger |1\rangle = \sqrt{2}\,|2\rangle$ & $A\sqrt{2}|2\rangle = 2\,|1\rangle$

$\Rightarrow A A^\dagger A A^\dagger |0\rangle = |0\rangle$

$A^2 A^{\dagger 2} |0\rangle = 2|0\rangle$

so,
$$\Delta E_0^{(1)} = C_0 (1 + 2)$$

$$\boxed{\Delta E_0^{(1)} = \frac{3}{4} \lambda \frac{\hbar^2}{m^2 \omega^2}}$$

GA 16.3

$$H' = \lambda X^4$$

$$\Delta E_0^{(1)} = \langle n=0 | \lambda X^4 | n=0 \rangle$$

$$X = \sqrt{\frac{\hbar}{2m\omega}}\left(A + A^{+}\right)$$

$$\Delta E_0^{(1)} = \lambda\left(\sqrt{}\right)^4 \langle 0 | \left(A + A^{+}\right)^4 | 0 \rangle$$

$$= C_0 \langle 0 | (A + A^{+})(A + A^{+})(A + A^{+})(A + A^{+}) | 0 \rangle$$

ONLY TERMS w/ EQUAL AMOUNTS OF RAISING & LOWERING WILL SURVIVE

$$\Delta E_0^{(1)} = C_0 \langle 0 | (A^2 + A^{+}A + AA^{+} + A^{+2})$$

$$\times (A^2 + A^{+}A + AA^{+} + A^{+2}) | 0 \rangle$$

~~C_0~~ ALSO, $A|0\rangle = 0$ SO TERMS w/ AN A ON RHS DO NOT SURVIVE

$$\Delta E_0^{(1)} = C_0 \langle 0 | (A^2 + A^{+}A + AA^{+} + A^{+2})(AA^{+} + A^{+2}) | 0 \rangle$$

#4

$$H' = -\frac{G m_e m_p}{r}$$

$$E_0^{(1)} = \left\langle 1s \left| -\frac{G m_e m_p}{r} \right| 1s \right\rangle$$

$$= -k \langle \psi_{100} | \tfrac{1}{r} | \psi_{100} \rangle$$

$$= -k/a_0 \quad \text{(SEE GA 12-36)}$$

$$\boxed{E_0^{(1)} = -\frac{G m_e m_p}{a_0}}$$

$$\frac{E_0^{(1)}}{E_0} = \frac{G m_e m_p}{(13.6\,eV)\,a_0}$$

$$\boxed{\frac{E_0^{(1)}}{E_0} = 7.4 \times 10^{-40}}$$

w/ $1\,eV = 1.6 \times 10^{-19}$ J