

**PHYS 5383**

Spring 2012

TE Coan

Due: 2 Feb '12 6:00 pm

Homework 1

1. Consider the operator  $\hat{C}$

$$\hat{C}\phi(x) = \phi^*(x)$$

- a. Is  $\hat{C}$  Hermitian? Note that you do not need to actually find  $\hat{C}^\dagger$ .
- b. What are the eigenfunctions of  $\hat{C}$ ?
- c. What are the eigenvalues of  $\hat{C}$ ?

2. The norm (or “length”) of a function or element in a Hilbert space is related to its inner product by the relation

$$(\text{norm of } \phi(x))^2 = \|\phi\|^2 = \langle \phi | \phi \rangle$$

The parallelogram law of geometry says that the sum of the square of the diagonals of a parallelogram (what’s that?) equals twice the sum of the squares of the sides. Show that this is true in Hilbert space; that is, if  $\psi$  and  $\phi$  are any two elements (say, eigenfunctions) of a Hilbert space, then

$$\|\psi + \phi\|^2 + \|\psi - \phi\|^2 = 2\|\psi\|^2 + 2\|\phi\|^2$$

This is another example of the similarity of “vector arrow space” you are used to in PHYS 1303 and 1304 and the much more abstract Hilbert space. It may help to draw a diagram of a parallelogram.

3. Consider the Hermitian operator  $\hat{H}$  that has the property

$$\hat{H}^4 |\phi\rangle = 1$$

- a. What are the eigenvalues of the operator  $\hat{H}$ ?
  - b. What are the eigenvalues of the operator  $\hat{H}$  if it is **not** restricted to be Hermitian?
4. An electron in an oscillating electric field is described by the Hamiltonian operator

$$\hat{H} = \frac{\hat{p}^2}{2m} - (eE_0 \cos \omega t)x.$$

- a. Calculate  $\frac{d}{dt}\langle x \rangle$ .
- b. Calculate  $\frac{d}{dt}\langle p \rangle$ .
- c. Calculate  $\frac{d}{dt}\langle H \rangle$ .

5. The (complex) exponentiation of the operator  $\hat{H}$  is defined to be  $e^{i\hat{H}} = \sum_{n=0}^{\infty} (i\hat{H})^n / n!$ . That is, you just behave as if  $i\hat{H}$  is a real number and use the Taylor series expansion for  $e^x$  for  $x$  close to zero. Show that if  $\hat{H}$  is Hermitian,  $(e^{i\hat{H}})^\dagger = e^{-i\hat{H}}$ .