#1 $\hat{C}\,\varphi(x) = \varphi^*(x)$

(a)

$\hat{C}^\dagger$ DEFINED BY: $\langle \varphi | \hat{C}^\dagger | \psi \rangle = \langle C\varphi | \psi \rangle$

<u>IF</u> $\hat{C}^\dagger$ HERMITIAN, $\hat{C}^\dagger = \hat{C}$

CHECK IT:

$$\langle \varphi | C | \psi \rangle \overset{?}{=} \langle C\varphi | \psi \rangle$$

$$\int \varphi^* \psi^* \, dx \overset{?}{=} \int (\varphi^*)^* \psi \, dx$$

$$\int \varphi^* \psi^* \, dx \overset{?}{=} \int \varphi \psi \, dx$$

<u>NOT</u> NECESSARILY ; $\hat{C}$ NOT HERMITIAN

(b) $\hat{C}\,\varphi(x) = \varphi^*(x)$

$C^2 \varphi(x) = \varphi(x)$

WRITE AS EIGENVALUE PROBLEM

$C^2 \varphi(x) = c^2 \varphi(x)$ w/ c = EIGENVALUE OF $\hat{C}$

$\Rightarrow c^2 = 1$, $c = \pm 1$

FOR $c = 1$: $C\varphi(x) = \varphi^*(x) = \varphi(x)$; $\varphi(x)$ REAL.

$c = -1$ $C\varphi(x) = \varphi^*(x) = -\varphi(x)$; $\varphi(x)$ PURELY IMAGINARY.

#1(b) CONT.

$\boxed{\begin{array}{l}\hat{C} \text{ EIGENFUNCTIONS: PURELY REAL} \\ \text{OR " IMAGINARY}\end{array}}$

(c) FROM ABOVE EIGENVALUE = +1
FOR PURELY REAL E.F.

EIGENVALUE = −1
FOR PURELY IMAGINARY E.F.

#2. $\|\psi+\varphi\|^2 = (\langle\psi| + \langle\varphi|)(|\psi\rangle + |\varphi\rangle)$

w/ $|\psi\rangle$ & $|\varphi\rangle$ IN HILBERT SPACE

$\|\psi + \varphi\|^2 = \langle\psi|\psi\rangle + \langle\psi|\varphi\rangle + \langle\varphi|\psi\rangle + \langle\varphi|\varphi\rangle$

DO NOT REQUIRE $\langle\varphi|\psi\rangle = 0$

$\|\psi - \varphi\|^2 = (\langle\psi| - \langle\varphi|)(|\psi\rangle - |\varphi\rangle)$

$= \langle\psi|\psi\rangle - \langle\varphi|\psi\rangle - \langle\psi|\varphi\rangle + \langle\varphi|\varphi\rangle$

ADDING.

SO

$$\boxed{\|\psi+\varphi\|^2 + \|\psi-\varphi\|^2 = 2\langle\psi|\psi\rangle + 2\langle\varphi|\varphi\rangle}$$

#3.

$$H^4 |\varphi\rangle = |\varphi\rangle$$

WRITE AS EIGENVALUE PROBLEM:

$$H^4 |\varphi\rangle = a^4 |\varphi\rangle$$

SO, $a^4 = 1 \Rightarrow \boxed{a = \pm 1.}$ a REAL. SINCE $\hat{H}$ HERMITIAN.

IF $\hat{H}$ NOT HERMITIAN,

$a^4 = 1$, BUT NOW, $\boxed{a = \pm 1, \pm i \text{ FOR } \hat{H} \neq \hat{H}^\dagger}$

#4.

(i) $\langle d\hat{Q}/dt \rangle = \langle \frac{\partial Q}{\partial t} \rangle + \frac{1}{i\hbar} \langle [Q, H] \rangle$

$$\frac{d}{dt}\langle x \rangle = \langle \partial x/\partial t \rangle + \frac{1}{i\hbar} \langle [x, H] \rangle$$

$$= 0 + \frac{1}{i\hbar} \langle [\hat{x}, \hat{p}^2/2m] \rangle$$

$$= \frac{1}{i\hbar} \cdot \frac{1}{2m} \langle [x, p^2] \rangle$$

$$= \frac{1}{2m} \frac{1}{i\hbar} \langle xpp - ppx \rangle$$

$$= \frac{1}{2m} \frac{1}{i\hbar} \langle xpp - pxp + pxp - ppx \rangle$$

$$= \frac{1}{2m} \frac{1}{i\hbar} \langle [x, p] p + p [x, p] \rangle$$

$$= \frac{1}{2m} \frac{1}{i\hbar} \langle i\hbar p + p i\hbar \rangle$$

$$\boxed{\frac{d}{dt}\langle x \rangle = \frac{1}{m} \langle p \rangle}$$

#4 (b)

$$\frac{d}{dt}\langle p\rangle = \langle \partial p/\partial t\rangle + \frac{1}{i\hbar}\langle [p, H]\rangle$$

$$= 0 + \frac{1}{i\hbar}\langle -eE_0 \cos\omega t\, [p, x]\rangle$$

$$\boxed{\frac{d}{dt}\langle p\rangle = +eE_0 \cos\omega t}$$

(c) $\frac{d}{dt}\langle \hat{H}\rangle = \langle \partial \hat{H}/\partial t\rangle + \frac{1}{i\hbar}\langle [\hat{H}, \hat{H}]\rangle$

$$\boxed{\frac{d\langle \hat{H}\rangle}{dt} = \langle \frac{\partial}{\partial t} H\rangle = +\omega eE_0 \sin\omega t \langle x\rangle}$$

TO MAKE FURTHER PROGRESS, NEED TO SOLVE FOR $\langle x(t)\rangle$. IN PRINCIPLE, THIS CAN BE DONE USING PTS (a) & (b), W/ SUITABLE BOUNDARY CONDITIONS.

(5) $e^{iH} = \{ 1 + iH + \frac{(iH)^2}{2!} + \frac{(iH)^3}{3!} + \cdots$

$(e^{iH})^\dagger = \{ \quad \}^\dagger$

$= 1 + (iH)^\dagger + \frac{[(iH)^2]^\dagger}{2!} + \frac{[(iH)^3]^\dagger}{3!} + \cdots$

$= 1 + H^\dagger i^\dagger + \frac{(iH)^\dagger (iH)^\dagger}{2!} + \frac{(iH)^\dagger (iH)^\dagger (iH)^\dagger}{3!} + \cdots$

$= 1 + (-iH) + \frac{(-iH)^2}{2!} + \frac{(-iH)^3}{3!} + \cdots$

$= e^{-iH}$

w/ $i^\dagger = -i$

$H^\dagger = H$

$\therefore \boxed{\{e^{iH}\}^\dagger = e^{-iH}}$