

PHYS 5383

Spring 2012

TE Coan

Due: 17 Feb '12 6:00 pm

Homework 2

1. The operator \hat{A} has only non-degenerate eigenvectors $\{\phi_n\}$ and eigenvalues $\{a_n\}$. What are the eigenvectors and eigenvalues of the inverse operator \hat{A}^{-1} ?
2. Recall from the faraway land of differential equations that the linear independence of two functions $s(x)$ and $v(x)$ may be specified by their Wronskian:

$$W(s, v) = \begin{vmatrix} s & v \\ s' & v' \end{vmatrix}.$$

If s and v are the solutions to a linear, second order differential equation and $W(s, v) \neq 0$ over some interval, then s and v are linearly independent over that interval. Using this criterion, establish that the two solutions to the radial Schrödinger in one dimension with an infinite spherical well and with $l = 0$ are independent over the size of the well. What value for W do you find in this case?

3. Here is a bit of review. At $t = 0$ it is known that of 1000 neutrons in a one-dimensional box of width 10^{-7} m, 100 have energy $4E_1$ and 900 have energy $225E_1$.

- a. Construct a wavefunction with these properties. (Coefficients of the eigenfunctions may be complex.)

- b. What is the density $\rho(x)$ of neutrons per unit length?

- c. How many neutrons are in the left hand side of the “box”?

4. Consider the situation where it is equally likely that an electron has momentum $\pm\mathbf{p}_0$. Measurement at a given instant of time finds the value $+\mathbf{p}_0$. A pesky student concludes that the electron must have had this value of momentum prior to the measurement. Is the student correct? Explain. **Briefly.**

5. As threatened in lecture, prove that the coefficient A for the function $u(r) = A \sin kr + B \cos kr$ that satisfies the radial equation for the infinite spherical well with $l = 0$ is $A = \sqrt{2/a}$, where a is the radius of the well.