#1

$$\hat{A}\varphi_n = a_n \varphi_n$$

$$\hat{A}^{-1}\hat{A}\varphi_n = A^{-1}(a_n \varphi_n)$$

$$\varphi_n = a_n A^{-1}\varphi_n$$

$$\Rightarrow \quad A^{-1}\varphi_n = \frac{1}{a_n}\varphi_n$$

So, $\boxed{\text{E. Functions of } \hat{A}^{-1} = \{\varphi_n\} \quad \text{w/ E. VALUES} = \{1/a_n\}}$

#2. FROM LECTURE & GRIFFITHS.

$$u(r) = A\sin kr + B\cos kr$$

$$\text{w/ } k = \frac{\sqrt{+2mE}}{\hbar}$$

Let $s = A\sin kr$ $\quad v = B\cos kr$

$s' = Ak\cos kr$ $\quad v' = -Bk\sin kr$

$\|W\| = sv' - vs'$

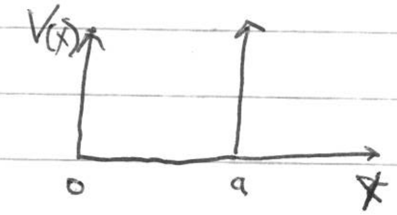
$$= A\sin kr\,(-Bk\sin kr) - B\cos kr\, Ak\cos kr$$

$$= -ABk\,(\sin^2 kr + \cos^2 kr)$$

$$\boxed{\|W\| = -ABk \neq 0}$$

SO, s & v ARE LINEARLY INDEPENDENT.

#3.



$\psi(x) = A \sin kx + B \cos kx$

BUT $\psi(0) = 0 \Rightarrow B = 0$

$\psi(x) = A \sin kx \quad$ w/ $\quad k = \sqrt{2mE}/\hbar$

BC @ $x = a \quad \psi(a) = A \sin ka = 0$

$\Rightarrow ka = n\pi \qquad n = 1, 2, 3, \ldots$

$k = n\pi/a$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\frac{\pi^2}{a^2} n^2$$

FROM ENERGIES $4E_1$ & $225 E_1$, $n = 2, 15$

NORMALIZATION $\quad \int_0^a A^2 \sin^2 kx\, dx = 1$

$$\int_0^a A^2\left[\frac{1 - \cos 2kx}{2}\right]dx = 1$$

$$\Rightarrow A = \sqrt{2/a}$$

EIGENFUNCTIONS $\quad \psi_2(x) = \sqrt{2/a}\, \sin k_2 x$

$$\psi_{15}(x) = \sqrt{2/a}\, \sin k_{15} x$$

w/ $\quad k_n = n\pi/a \quad , a = 10^{-7}\,m$

So,

(a)

$$\boxed{\psi(x) = c_2\,\varphi_2(x) + c_{15}\,\varphi_{15}(x) \quad \text{w/ } c_2 = 10/\sqrt{1000}\,,\ c_{15} = 30/\sqrt{1000}}$$

(b) $\rho(x) = |\psi(x)|^2 \cdot N_n$. NOTE $\int_0^a |\psi(x)|^2 dx = 1$

$$\boxed{\rho(x) = \left[c_2^2\,\varphi_2^2(x) + c_{15}^2\,\varphi_{15}^2(x) + 2c_2 c_{15}\,\varphi_2(x)\,\varphi_{15}(x)\right] N_n}$$

w/ $N_n = 10^3$

(c) $\int_0^a \rho(x)\,dx = N_n$

NOTE THAT BOTH $\varphi_2(x)$ & $\varphi_{15}(x)$ ARE SYMMETRIC ABOUT $x = a/2$.
HENCE $\rho(x)$ SYMMETRIC ABOUT $x = a/2$
SO, 500 NEUTRONS IN LHS OF BOX.

#4 $\underline{\underline{\text{No.}}}$ $\psi(x)$ $\underline{\text{BEFORE}}$ M'MENT IS

$$\psi(x) = \frac{\varphi_{+p}(x) \pm \varphi_{-p}(x)}{\sqrt{2}},$$

i.e., $\underline{\text{NOT}}$ $\varphi_{+p}(x)$ $\underline{\underline{\text{OR}}}$ $\varphi_{-p}(x)$ ALONE,

AND CERTAINLY NOT JUST $\varphi_{+p}(x)$.

#5 $\int_0^a u^2(r)\,dr = 1.$ w/ $u(r) = A\sin kr + B\cos kr$

$= A\sin kr$

since $u(0) = 0$

$$\int_0^a A^2 \sin^2 kr \, dr = 1$$

$$\int_0^a A^2 \left(\frac{1 - \cos 2kr}{2}\right) dr = 1$$

$$A^2 \left[\frac{r}{2} + \frac{\sin 2kr}{4}\right]\Bigg|_0^a = 1$$

$A^2 \frac{a}{2} = 1$ SINCE $\sin 2ka = 0$

$$\boxed{A = \sqrt{2/a}}$$