(a)

~~#~~ SEEK TO PROVE $\int (P\psi)^* \psi \, d^3r = \int \psi^* P \psi \, d^3r$

(1) OR, IN BRA-KET NOTATION $\langle Pr | r \rangle = \langle r | Pr \rangle$

WE KNOW HOW $\hat{P}$ ACTS:

$$\hat{P}\,\psi(r) = \psi(-r)$$

$$\hat{P}|r\rangle = |-r\rangle$$

NOW,

$$\int \psi^*(r)\, \psi(-r)\, d^3r = \int \psi^*(-r)\, \psi(r)\, d^3r \qquad (2)$$

THIS IS EASIEST TO SEE IN SPHERICAL COORDINATES, WHERE $r \geq 0$ AND INTEGRAND IS INDEPENDENT OF HOW YOU ROTATE YOUR COORD SYSTEM.

CONTINUING

$$\int \psi^*(-r)\, \psi(r)\, d^3r = \int (P\psi)^* \psi \, d^3r$$

HENCE FROM (2), $\int (P\psi)^* \psi \, d^3r = \int \psi^*(r)\, \psi(-r)\, d^3r$

$$\boxed{\begin{array}{c} \langle Pr | r \rangle = \langle r | Pr \rangle \\ \therefore\ \hat{P} \text{ IS HERMITIAN} \end{array}}$$

#1 (b) $P|r\rangle = |-r\rangle$

$P^2|r\rangle = |r\rangle$ EIGENVALUE EQN ?

$\Rightarrow P^2 = 1 \,,\; P = \pm 1$

(c) $\hat{P}^\dagger = \hat{P}$ SINCE $\hat{P}$ IS HERMITIAN AS SHOWN IN PT (a).

$P|r\rangle = |-r\rangle$

$P\,P|r\rangle = P|-r\rangle = |r\rangle$

$P^\dagger P|r\rangle = |r\rangle$

$\Rightarrow \boxed{P^\dagger P = 1 \quad \therefore P^\dagger = P^{-1}}$

#2. $PAP = A$ $\quad$ A EVEN $\quad$ (1)

FROM ABOVE, $P^{\dagger} = P$ & $P^{\dagger} = P^{-1}$

$$P^{-1} P A P = P^{-1} A$$

$$AP = P^{\dagger} A = PA$$

$$AP - PA = 0$$

$$\boxed{[A, P] = 0}$$

$PBP = -B$ $\quad$ B ODD

$\cancel{PBP = -B}$

$$PBPP^{-1} = -BP^{-1} = -BP^{\dagger} = -BP$$

$$PB = -BP$$

$$PB + BP = 0$$

$$\boxed{\therefore \{P, B\} = 0}$$

#2 (b) DISTINGUISH THE TWO POSSIBILITIES

$$PRP = +R$$

OR

$$PRP = -R$$

WHICH IS IT?

$$PRP|r\rangle = PR|-r\rangle \qquad \text{w/ } R|r\rangle = r|r\rangle$$
$$= P(-r)|-r\rangle$$
$$= -r\,P|-r\rangle$$
$$PRP|r\rangle = -r|r\rangle$$

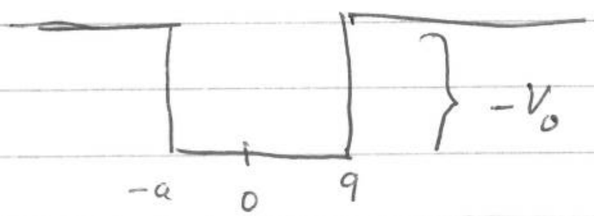
$$PRP|r\rangle = -R|r\rangle$$

$$\boxed{\therefore\ \hat{R} \text{ IS ODD}}$$

(c) CHANGING VARIABLES FROM $\underset{\sim}{r}$ to $\underset{\sim}{p}$
AND USING THE SAME MATH
YIELDS,

$$\boxed{\hat{p} \text{ IS ODD}}$$

#3.




SIMILAR TO PROBLEM IN TEXT.

$$\left[-\frac{\hbar^2}{2m} d^2/dx^2 + V(x)\right]\psi = E\psi$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - V_0\right]\psi = E\psi \qquad -a \le x \le a$$

$$\frac{d^2}{dx^2}\psi = -\frac{2m}{\hbar^2}(V_0 + E)\psi$$

$$= -q^2\psi \qquad q = \frac{\sqrt{2m(V_0+E)}}{\hbar}$$

$$\Rightarrow \psi = A\sin qx + B\cos qx \qquad -a \le x \le a$$

For $|x| > a$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$$

$$\frac{d^2}{dx^2}\psi = k^2\psi \qquad \text{w/ } k = \frac{\sqrt{-2mE}}{\hbar}$$

$\;$ & $E < 0$

$$\psi(x) = Ce^{-kx} + De^{+kx}$$

$|x| > a$.

#3. $\psi(x) = A \sin qx \qquad -a \le x \le a$

& WE WANT ODD SOLS.

$= De^{-kx} \qquad x \ge a$

ALSO, $\psi(-x) = -\psi(x)$ SINCE WE WANT ODD SOLS

$\psi$ CONTINUITY @ $x=a$:

$$De^{-ka} = A \sin qa \qquad (10)$$

$\psi'$ CONTINUITY @ $x=a$ $\quad -Dke^{-ka} = Aq \cos qa \qquad (11)$

DIVIDING EQ (11) BY EQ (10) YIELDS

$$-k = q \cot qa$$

$$-ka = qa \cot qa \qquad (12)$$

w/ $y \equiv qa$, (12) BECOMES

$$-ka = y \cot y$$

$$ka = -y \cot y \qquad (13)$$

NOW, $k^2a^2 = \dfrac{-2mEa^2}{\hbar^2} = \dfrac{2mV_0a^2}{\hbar^2} - \dfrac{2m(V_0+E)a^2}{\hbar^2}$

$$= R^2 - q^2a^2$$

$$k^2a^2 = R^2 - y^2 \qquad (14)$$

#3 SUBSTITUTING (14) INTO (13) YIELDS

$$\boxed{\sqrt{R^2 - y^2} = -y \cot y}$$

w/ $R^2 = \dfrac{2mV_0 a^2}{\hbar^2}$ & $y = ga$

#4 (a) USE PROCEDURE YOU USED W/ QUESTION 3. SINCE $l=0$ AND POTENTIALS ARE SIMILAR, WE ~~HAVE~~ HAVE

$u(r) = A\sin qr + B\cos qr$   $0 \le r \le a$

w/ $q^2 = \frac{2m(V_0+E)}{\hbar^2}$

$E<0$

$u(r=0)=0 \Rightarrow B=0$

For $r>a$.

$u(r) = Ce^{-kr}$   w/ $k^2 = -2mE/\hbar^2$

$E<0$

AGAIN, WE NEED CONTINUITY AT $r=a$ FOR BOTH $u(r)$ & $u'(r)$!

$A\sin qa = Ce^{-ka}$
$Aq\cos qa = -Cke^{-ka}$

$-k = q\cot(qa)$

$+ka = -qa\cot(qa)$

(5) $\sqrt{R^2-y^2} = -y\cot(qa)$   w/ $y=qa$
$R^2 = 2mV_0a^2/\hbar^2$

WE ALSO KNOW THAT THERE IS A BOUND STATE w/ $E = -2.23$ MeV, AND THAT $a \simeq 2.4$ Fermi.

CONVERT TO SENSIBLE UNITS USING CONVERSION $\hbar c = 200$ MeV- Fermi.

FINALLY, $\mu \simeq 480$ MeV$/c^2$

$$R^2 = \frac{2mV_0 a^2}{\hbar^2} = \frac{2 * 480 \times 2.4^2}{200^2} V_0$$

$$= 0.138\, V_0$$

$$y^2 = \frac{2mV_0 a^2}{\hbar^2} - \frac{2m|E|a^2}{\hbar^2} \Rightarrow y = [0.1382\, V_0 - 0.3083]^{1/2}$$

$$R^2 - y^2 = \frac{+2m|E|a^2}{\hbar^2} = 0.3083$$

$$\sqrt{R^2 - y^2} = 0.5552$$

COMBINING, $\sqrt{R^2 - y^2} = -y \cot y$

$$0.5552 - [0.1382\, V_0 - 0.3083]^{1/2}$$
$$* \cot [0.1382\, V_0 - 0.3083]^{1/2}$$

SOLVE FOR $V_0$ USING MATHEMATICA.

$V_0 = 27.3$ MeV

#4

```
FindRoot[0.5552 + Sqrt[0.1382 x - 0.3083] Cot[Sqrt[0.1382 x - 0.3083]] == 0, {x, 30}]

{x → 27.2846}

In[1]:= Show[Plot[ -Sqrt[0.1382 x - 0.3083] Cot[Sqrt[0.1382 x - 0.3083]   ], {x, 0, 30}],
  Plot[0.5552, {x, 0, 30}]]
```

Out[1]=