#1.

$$(\hat{\sigma}\cdot \underset{\sim}{A})(\hat{\sigma}\cdot \underset{\sim}{B}) = \sigma_i A_i \sigma_j B_j$$

$$= A_i B_j \sigma_i \sigma_j$$

$$= A_i B_j \left[\delta_{ij} \mathbb{1} + i\, \epsilon_{ijk}\, \sigma_k \right]$$

$$\boxed{\hat{\sigma}\cdot \underset{\sim}{A}\; \hat{\sigma}\cdot \underset{\sim}{B} = \underset{\sim}{A}\cdot \underset{\sim}{B} + i\,(\underset{\sim}{A}\times \underset{\sim}{B})\cdot \hat{\sigma}}$$

#2. TWO COMPONENT SPINOR, SPIN = $\frac{1}{2}\hbar$

NOTE THAT OPERATOR IS OF FORM $\hat{S}\cdot\hat{n}$, AS IN RECENT TEST.

ROTATE YOUR COORDINATE SYSTEM UNTIL YOUR Z-AXIS IS ALIGNED W/ $\hat{n}$ DIRECTION.

THEN SINCE IN GENERAL

$|\langle \uparrow | \text{SPIN} \rangle|^2$ = PROBABILITY OF SPIN UP

$$\left| (1\ 0) \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|^2 = 4/5$$

$|\langle \downarrow | \text{SPIN} \rangle|^2$ = PROB OF SPIN DOWN

$$\left| (0\ 1) \frac{1}{\sqrt{5}} \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right|^2 = 1/5$$

$\boxed{\text{PROB OF MEASURING } -\hbar/2 = 20\%}$

#3. $$\boxed{\left|\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{6}}\left|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle|1\,-1\rangle - \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2}\,-\tfrac{1}{2}\right\rangle|1\,0\rangle + \sqrt{\tfrac{1}{2}}\left|\tfrac{3}{2}\,-\tfrac{3}{2}\right\rangle|1\,1\rangle}$$

#4(a). SUBSTITUTE THE $\hat{S}_+$ OPERATOR FOR $\hat{S}_z$ IN THE EXAMPLE IN THE PROBLEM.

NOTE $S_+|1\,1\rangle = 0$

$S_+|1\,0\rangle = \sqrt{2}\,\hbar\,|1\,1\rangle$

$S_+|1\,-1\rangle = \sqrt{2}\,\hbar\,|1\,0\rangle$

$$\Rightarrow \boxed{\hat{S}_+ = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}}$$

SIMILARLY,

$S_-|1\,1\rangle = \sqrt{2}\,\hbar\,|1\,0\rangle$

$S_-|1\,0\rangle = \sqrt{2}\,\hbar\,|1\,-1\rangle$

$S_-|1\,-1\rangle = 0$

OR $S_- = S_+^\dagger$

$$\text{BOTH} \Rightarrow \boxed{S_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}}$$

#4(b) $S_+ = S_x + i S_y$

$S_- = S_x - i S_y$

$\Rightarrow S_x = \frac{1}{2}(S_+ + S_-)$

$S_y = \frac{1}{2i}(S_+ - S_-)$

FROM ABOVE,

$$\boxed{S_x = \hbar \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 0 \end{pmatrix}}$$

$$\boxed{S_y = \hbar \begin{pmatrix} 0 & -i/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & i/\sqrt{2} & 0 \end{pmatrix}}$$

#5

(a) $A(t) = a(t)\, e^{i\omega_0 t}$

$$\dot{A}(t) = \dot{a}(t)\, e^{i\omega_0 t} + a(t)\,(i\omega_0)\, e^{i\omega_0 t}$$

$$i\dot{A}(t) = i\dot{a}(t) e^{i\omega_0 t} - \omega_0\, a(t)\, e^{i\omega_0 t}$$

$$= i\dot{a}(t)\, e^{i\omega_0 t} - \omega_0 A(t)$$

$$= -\omega_0 A(t) + (\omega_0 a(t) + \omega_1 b(t) \cos\omega t)\, e^{i\omega_0 t}$$

$$= -\omega_0 A(t) + \omega_0 A(t) + \omega_1 b(t) \cos\omega t\; e^{i\omega_0 t}$$

$$\boxed{i\dot{A}(t) = \omega_1 B(t) \cos\omega t \;\; e^{2i\omega_0 t}} \qquad (1)$$

SIMILARLY, $\dot{B}(t) = \dot{b}(t)\, e^{-i\omega_0 t} + b(t)\,(-i\omega_0)\, e^{-i\omega_0 t}$

$$i\dot{B}(t) = i\dot{b}(t)\, e^{-i\omega_0 t} + \omega_0\, b(t)\, e^{-i\omega_0 t}$$

$$i\dot{B}(t) = +\omega_0 b(t) e^{-i\omega_0 t} + (\omega_1 a(t) \cos\omega t - \omega_0 b(t))\, e^{-i\omega_0 t}$$

$\cancel{i\dot{B}(t) =}$

$$\boxed{i\dot{B}(t) = \omega_1 A(t) \cos\omega t \;\; e^{-2i\omega_0 t}} \qquad (2)$$

#5(b). USE THE ROTATING WAVE APPROXIMATION,

$$\cos\omega t\, e^{2i\omega_0 t} \simeq \frac{1}{2} e^{i(2\omega_0-\omega)t}$$

REWRITE EQ (1) FROM (a) AS

$$\boxed{i\dot{A}(t) \simeq \frac{1}{2}\omega_1 B(t)\, e^{i(2\omega_0-\omega)t}} \qquad (3)$$

REWRITE EQ (2) FROM (a) AS.

$$\boxed{i\dot{B}(t) \simeq \frac{1}{2}\omega_1 A(t)\, e^{-i(2\omega_0-\omega)t}} \qquad (4)$$

(c) DIFFERENTIATE EQ. (3) AND USE (4) TO OBTAIN

$$d^2A/dt^2 = (2\omega_0-\omega)\, i\, dA/dt - \left(\omega_1/2\right)^2 A(t) \qquad (5)$$

TRY $A(t) = A_0 e^{i\Omega t}$

$$\dot{A}(t) = i\Omega A_0 e^{i\Omega t}$$

$$\ddot{A}(t) = -\Omega^2 A_0 e^{i\Omega t}$$

SUB THESE RESULTS INTO (5):

$$-\Omega^2 = \frac{-\omega_1^2}{4} - (2\omega_0-\omega)\Omega$$

$$\Omega^2 - (2\omega_0 - \omega)\Omega - (\omega_1/2)^2 = 0$$

$$\Rightarrow \Omega = \frac{+(2\omega_0-\omega) \pm \left[(2\omega_0-\omega)^2 + \omega_1^2\right]^{1/2}}{2}$$

$$\boxed{\therefore \Omega = \left(\omega_0 - \frac{\omega}{2}\right) \pm \left[\left(\omega_0 - \frac{\omega}{2}\right)^2 + \omega_1^2/4\right]^{1/2}}$$

(d) $A(t) = A_+ e^{i\Omega_+ t} + A_- e^{-i\Omega_- t}$

SINCE $i\dot{A}(t) = \frac{\omega_1}{2} B(t) e^{i(2\omega_0-\omega)t}$

$$B(t) = \frac{2i}{\omega_1}\dot{A}(t)\, e^{-i(2\omega_0-\omega)t}$$

$$= \frac{2i}{\omega_1} e^{-i(2\omega_0-\omega)t}\left[A_+ i\Omega_+ e^{i\Omega_+ t} + iA_-\Omega_- e^{i\Omega_- t}\right]$$

$$= -\frac{2}{\omega_1} e^{-i(2\omega_0-\omega)t}\left[\underset{①}{A_+\Omega_+ e^{i\Omega_+ t}} + \underset{②}{A_-\Omega_- e^{i\Omega_- t}}\right] \quad (8)$$

NOW,

TERM ① $= e^{-i(2\omega_0-\omega)t} e^{i\Omega_+ t} = e^{i[(\omega_0-\frac{\omega}{2}) + \surd - 2\omega_0 + \omega]t}$

$$= e^{i(-\omega_0 + \frac{\omega}{2} + \surd)t}$$

① $= e^{-i[(\omega_0 - \frac{\omega}{2}) - \surd]t} = e^{-i\Omega_- t}$

$$\text{w/ } \surd = \left[\left(\omega_0 - \frac{\omega}{2}\right)^2 + \omega_1^2/4\right]^{1/2}$$

LIKEWISE, TERM ② CAN BE WRITTEN AS

$$② = e^{-i(2\omega_0-\omega)t}\, e^{i\Omega_- t}$$

$$= e^{i\left[\omega_0 - \frac{\omega}{2} - \sqrt{} - 2\omega_0 + \omega\right]t}$$

$$= e^{i\left[-\omega_0 + \frac{\omega}{2} - \sqrt{}\right]t}$$

$$= e^{-i\left[\omega_0 - \frac{\omega}{2} + \sqrt{}\right]t} = e^{-i\Omega_+ t}$$

COLLECTING RESULTS AND SUB'ING INTO EQ (8) YIELDS

$$\boxed{B(t) = \frac{-2}{\omega_1}\left(\Omega_+ A_+ e^{-i\Omega_- t} + \Omega_- A_- e^{-i\Omega_+ t}\right)}$$

(e) $|\langle\downarrow|\,\mathrm{SPIN}(t)\rangle|^2$ = PROBABILITY OF MEASURING SPIN DOWN

$$= \left|\,(0\;1)\begin{pmatrix} a(t) \\ b(t)\end{pmatrix}\right|^2 = |b(t)|^2$$

w/ $b(t) = B(t)\, e^{-i\omega_0 t}$

$$|b(t)|^2 = |B(t)|^2$$

$$|B(t)|^2 = \left(\frac{2}{\omega_1}\right)^2 \left| \frac{\Omega_+\Omega_-}{\Omega_- - \Omega_+} e^{+i\Omega_- t} - \frac{\Omega_-\Omega_+}{\Omega_- - \Omega_+} e^{+i\Omega_+ t} \right|^2$$

$$= \frac{4}{\omega_1^2}\left[\frac{\Omega_+\Omega_-}{\Omega_- - \Omega_+}\right]^2 \left| e^{+i\Omega_- t} - e^{i\Omega_+ t} \right|^2 \qquad (12)$$

BUT

$$|\quad|^2 = \left| e^{+i\Omega_- t} (1 - e^{+i(-\Omega_- + \Omega_+)t} \right|^2$$

$$= \left| 1 - e^{+i(-\Omega_- + \Omega_+)t} \right|^2$$

$$= \left| 1 - e^{+i\alpha t} \right|^2 \qquad \text{w/ } \alpha \equiv \Omega_+ - \Omega_-$$

$$= \left| e^{i\alpha t/2} \left(e^{-i\alpha t/2} - e^{+i\alpha t/2} \right) \right|^2$$

$$= \left| e^{i\alpha t/2}\, 2i \sin \alpha t/2 \right|^2$$

$$= 4 \sin^2 \alpha t/2 = 4\,(1 - \cos \alpha t)$$

$$|\,|^2 = 4\left[1 - \cos(\Omega_+ - \Omega_-)t\right]$$

SUB THIS INTO EQ (12):

$$P(t) = |B(t)|^2 = \frac{8}{\omega_1^2}\left(\frac{\Omega_+\Omega_-}{\Omega_- - \Omega_+}\right)^2 \left(1 - \cos(\Omega_+ - \Omega_-)t\right)$$

#5 (f).

AT $\omega = 2\omega_0$:

$$\Omega_+ = \omega_1/2$$

$$\Omega_- = -\omega_1/2$$

SO,

$$P(t) = \frac{8}{\omega_1^2}\left(\frac{\omega_1^2/4}{-\omega_1}\right)^2 \left(1 - \cos(-\omega_1)t\right)$$

$$\boxed{P_{RES}(t) = \frac{1}{2}\left(1 - \cos \omega_1 t\right)}$$