#1 (a) RECAST THE OPERATOR $\hat{\sigma}_1 \cdot \hat{\sigma}_2$

$$\hat{S}^2 = \left(\frac{\hbar}{2}\hat{\sigma}_1 + \frac{\hbar}{2}\hat{\sigma}_2\right)^2$$

$$= \frac{\hbar^2}{4}\sigma_1^2 + \frac{\hbar^2}{4}\sigma_2^2 + \frac{\hbar^2}{2}\hat{\sigma}_1 \cdot \hat{\sigma}_2$$

$$= \hat{S}_1^2 + \hat{S}_2^2 + \frac{\hbar^2}{2}\hat{\sigma}_1 \cdot \hat{\sigma}_2$$

$$\Rightarrow \hat{\sigma}_1 \cdot \hat{\sigma}_2 = \frac{2}{\hbar^2}\left[\hat{S}^2 - \hat{S}_1^2 - \hat{S}_2^2\right]$$

$$= \frac{2}{\hbar^2}\left[\hbar^2 s(s+1) - \frac{\hbar^2}{4}s_1(s_1+1) - \frac{\hbar^2}{4}s_2(s_2+1)\right]$$

$$\boxed{\hat{\sigma}_1 \cdot \hat{\sigma}_2 = 2s(s+1) - 3}$$

FOR SINGLET $(s=0)$ $\hat{\sigma}_1 \cdot \hat{\sigma}_2 = -3$

FOR TRIPLET $\hat{\sigma}_1 \cdot \hat{\sigma}_2 = +1$

TAKE HINT AND CHOOSE $\hat{e} \parallel \hat{z}$.

FIRST TERM IN S_{12} BECOMES

$$3(\hat{\sigma}_1 \cdot \hat{e})(\hat{\sigma}_2 \cdot \hat{e}) = 3\sigma_{1z}\sigma_{2z}$$

NOW, $3\sigma_{1z}\sigma_{2z}\chi_{SINGLET} = 3\sigma_{1z}\sigma_{2z} \cdot \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right]$

$$= -\frac{3}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right]$$

$$= -3\chi_{SINGLET}.$$

$$\boxed{3\sigma_{1z}\sigma_{2z}\chi_{SINGLET} = -3\chi_{SINGLET}}$$

WHERE I HAVE USED $\sigma_z|\uparrow\rangle = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$$\sigma_z|\downarrow\rangle = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\begin{pmatrix}0\\1\end{pmatrix}$$

COMBINING RESULTS:

$$S_{12}\chi_{SINGLET} = \left[3(\sigma_1\cdot\hat{e})(\sigma_2\cdot\hat{e}) - \sigma_1\cdot\sigma_2\right]\chi_{SINGLET}$$

$$= -3\chi_{SINGLET} + 3\chi_{SINGLET}$$

$$\boxed{S_{12}\chi_{SINGLET} = 0}$$

(b) FOR TRIPLET $S = 1\hbar$
$S_z = \pm\hbar, \phi.$

S_z	$\hat{\sigma}_1\cdot\hat{\sigma}_2 = 2S(S+1) - 3$	$3\sigma_{1z}\sigma_{2z}$
$+\hbar$	$4-3=1$	3
$-\hbar$	$4-3=1$	3
ϕ	1	-3

AGAIN, $S_{12} = 3\sigma_{1z}\sigma_{2z} - \hat{\sigma}_1\cdot\hat{\sigma}_2$

FOR $S_z = \pm\hbar$

$$S_{12}\,\chi_{TRIPLET}(m=\pm\hbar) = 3\chi_{TRIP}(m=\pm\hbar) - \chi_{TRIP}(m=\pm\hbar)$$

$$\boxed{S_{12}\,\chi_{TRIP}(m=\pm\hbar) = 2\chi_{TRIP}(m=\pm\hbar)}$$

FOR $S_z = 0\hbar$

$$S_{12}\,\chi_{TRIP}(m=0) = -3\chi_{TRIP}(m=0) - \chi_{TRIP}(m=0)$$

$$\boxed{S_{12}\,\chi_{TRIP}(m=0) = -4\chi_{TRIPLET}(m=0)}$$

COMBINING

$$(S_{12}-2)(S_{12}+4)\chi_{TRIPLET} = 0$$

#2 (a). IN A SINGLET STATE THE SPINS ARE <u>ANTI-PARALLEL</u> SO

$\boxed{\text{PROB OF BOTH SPINS "UP"} = 0}$

(b). THE KEY HERE IS TO EXPRESS SINGLET STATE IN TERMS OF EIGEN SPINORS OF S_x & S_y.

$\boxed{\chi_+^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}}$ FROM TEXT SPIN "UP" ALONG X

$\boxed{\chi_-^{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}}$ SPIN DOWN ALONG X.

FOR EIGENSPINORS FOR S_y.

$$S_y\,\chi_+^{(y)} = +\frac{\hbar}{2}\chi_+^{(y)}$$

$$\frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = \frac{\hbar}{2}\begin{pmatrix}a\\b\end{pmatrix}$$

$$-ib = a$$

$$ia = b$$

$\boxed{\chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}}$

SIMILARLY,

$\boxed{\chi_-^{(y)} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}}$

$$\chi_{\text{SINGLET}} = \frac{1}{\sqrt{2}}\left[\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2\right]$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_1 \begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1 \begin{pmatrix}1\\0\end{pmatrix}_2\right]$$

$$\begin{pmatrix}1\\0\end{pmatrix}_1 = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}\right]$$

$$\boxed{\begin{pmatrix}1\\0\end{pmatrix}_1 = \frac{1}{\sqrt{2}}\left[\xi_+ + \xi_-\right]}$$

w/ ξ_+ SPIN UP ALONG Y FOR PART (1)

ξ_- = SPIN DOWN ALONG Y

$$\begin{pmatrix}0\\1\end{pmatrix}_1 = \frac{-i}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}\right]$$

$$\boxed{\begin{pmatrix}0\\1\end{pmatrix}_1 = \frac{-i}{\sqrt{2}}\left[\xi_+ - \xi_-\right]}$$

EXPRESS $\begin{pmatrix}1\\0\end{pmatrix}$ & $\begin{pmatrix}0\\1\end{pmatrix}$ IN TERMS OF EIGENSPINORS ALONG X-DIRECTION

$$\begin{pmatrix}1\\0\end{pmatrix}_2 = \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right]$$

$$\boxed{\begin{pmatrix}1\\0\end{pmatrix}_2 = \frac{1}{\sqrt{2}}\left[\eta_+ + \eta_-\right]}$$

$\eta_\pm$ = SPINOR ALONG $\pm X$.

$$\boxed{\begin{pmatrix}0\\1\end{pmatrix}_2 = \frac{1}{\sqrt{2}}\left[\eta_+ - \eta_-\right]}$$

II 2 (b) NOW WE WRITE SINGLET STATE

$$\chi_{\text{SINGLET}} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}_1 \begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1 \begin{pmatrix}1\\0\end{pmatrix}_2\right]$$

$$= \frac{1}{\sqrt{2}}\Big[\frac{1}{\sqrt{2}}(\xi_+ + \xi_-)\cdot\frac{1}{\sqrt{2}}(\eta_+ - \eta_-)$$

$$- \frac{i}{\sqrt{2}}(\xi_+ - \xi_-)\cdot\frac{1}{\sqrt{2}}(\eta_+ + \eta_-)\Big]$$

THIS IS AN EXPANSION IN TERMS OF EIGENSPINORS ALONG y & x. LOOK FOR TERM $\xi_+\eta_+$ — THIS IS SPIN "UP" ALONG y AND SPIN "UP" ALONG x. FIND ~~THE~~ ITS COEFFICIENT AND SQUARE IT!

$\xi_+\eta_+$ TERM : $\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\xi_+\eta_+ + \frac{i}{\sqrt{2}}\xi_+\eta_+\right] = \frac{1}{2\sqrt{2}}(1+i)\xi_+\eta_+$

$$\text{PROB} = \left|\frac{1}{2\sqrt{2}}(1+i)\right|^2$$

$$= \frac{2}{4\cdot 2}$$

$$\boxed{\text{PROB} = 1/4}$$

#3. (a) $V(r) = V_1(r) + V_2(r)\left[\frac{3\,\sigma_1\cdot\hat{r}\;\sigma_2\cdot\hat{r}}{r^2} - \hat{\sigma}_1\cdot\hat{\sigma}_2\right]$

$$- V_3(r)\,\hat{\sigma}_1\cdot\hat{\sigma}_2$$

$$= V_1(r) + V_2(r)\left(S_{12}\right) + V_3\,\hat{\sigma}_1\cdot\hat{\sigma}_2$$

$$= V_1(r) + V_2(r)\,S_{12} + V_3(r)\underbrace{\left[2s(s+1) - 3\right]}_{\text{FROM PROB \#1, pt. B}}$$

FOR SINGLET $S_{12} = 0$, PROB #1.

$$\boxed{V(r) = V_1(r) - 3V_3(r)}$$

ALSO FROM PROB #1

(b)

FOR TRIPLET STATE

$$V(r) = V_1(r) + V_2(r)\left[3\sigma_{1z}\sigma_{2z} - 2s(s+1) + 3\right]$$

$$+ V_3(r)\left(2s(s+1) - 3\right)$$

$$V(r) = V_1(r) + V_2(r)\left[3\sigma_{1z}\sigma_{2z} - 1\right] + V_3(r)$$

ASSUMING EQUAL ~~PARTS~~ PROBABILITY FOR $m = 0, \pm 1\,\hbar$,

$$\boxed{V(r) = V_1(r) - V_2(r) + V_3(r)}$$

#4(a) A measurement of S_x will yield same eigenvalues as a measurement of S_z.

$$\boxed{\text{POSSIBLE VALUES OF } S_x = 0, \pm\hbar}$$

(b). FIND EIGENSPINOR FOR CASE $m_x = -\hbar$.

$$S_x \chi_-^{(x)} = -\hbar\, \chi_-^{(x)}$$

FROM HW 4, $S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

LET $\chi_-^{(x)} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$S_x \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\hbar \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\hbar \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\Rightarrow \frac{1}{\sqrt{2}} b = -a$$

$$\frac{1}{\sqrt{2}}(a+c) = -b$$

$$\frac{1}{\sqrt{2}} b = -c$$

SO,

$$\left.\begin{aligned} a &= c \\ \frac{1}{\sqrt{2}}\, 2a &= -b \end{aligned}\right\} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -b/\sqrt{2} \\ b \\ -b/\sqrt{2} \end{pmatrix}$$

#4(b) NORMALIZING, i.e., $|a|^2 + |b|^2 + |c|^2 = 1$

$$\boxed{\chi_-^{(x)} = \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}}$$

$$\langle S_z \rangle = \chi_-^{(x)\dagger} S_z \chi_-^{(x)}$$

$$= \frac{1}{2}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

$$= \frac{\hbar}{4}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\boxed{\langle S_z \rangle = 0}$$

$$\langle S_z^2 \rangle = \frac{1}{2}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

$$= \frac{\hbar^2}{4}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

$$= \frac{\hbar^2}{4}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

$$\boxed{\langle S_z^2 \rangle = \frac{\hbar^2}{2}}$$

#4. (c)

$$\langle S_y \rangle = \chi_-^{(x)\dagger} \; S_y \; \chi_-^{(x)}$$

$$= \frac{1}{2}\begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

w/ $S_y = (S_+ - S_-)/2i$, SEE HW 4,

$$\langle S_y \rangle = \frac{\hbar}{4} \begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \begin{pmatrix} -i\sqrt{2} \\ 0 \\ i\sqrt{2} \end{pmatrix}$$

$$\boxed{\langle S_y \rangle = 0}$$

$$\langle S_y^2 \rangle = \frac{\hbar^2}{8} \begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ \sqrt{2} \\ -1 \end{pmatrix}$$

$$\langle S_y^2 \rangle = \frac{\hbar^2}{8} \begin{pmatrix} -1 & \sqrt{2} & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 2\sqrt{2} \\ 0 \end{pmatrix}$$

$$\boxed{\langle S_y^2 \rangle = \hbar^2/2}$$