Lecture 14 Review

Chaos Intro w/ driven, damped pendulum.

Chaos Identification

- *Chaotic* motion is motion w/o any apparent regularity.
- Chaotic motion is NOT random motion.
- Random motion means you cannot predict future motion from present, even in principle.

- However, relevant <u>chaotic</u> ODE tells you how to get from present to future.
- IF you start w/ <u>identical</u> ICs, you <u>always</u> get the same final state.
- It is extreme sensitivity of a chaotic ODE to initial conditions that makes <u>practical</u> prediction of <u>far</u> future motion impossible.
- Change ICs slightly for chaotic system, very different final state.

Consider the example of the logistic map.

Chaos Identification (2)

$$x_{N+1} = \alpha x_N(1 - x_N)$$

Set $\alpha = 4.0$

Pick x_1 = 0.700 000 000

= 0.700 000 001

two different ICs

Find iteration N where the 2 solutions have clearly diverged.

N = ??

Chaos Identification (3)

$$x_{N+1} = \alpha x_N(1 - x_N)$$

Suppose difference between 2 sols, Δ, doubles every iteration.

After N iterations: $\Delta = 2^N = e^{N \ln 2}$

For final Δ ~ 1: $2^N 10^{-8} \sim 1$

$\Rightarrow N = 27$

Starting difference between ICs

Consider 2 initial states:

x_0

$x_0 + \varepsilon$ $\varepsilon << 1$

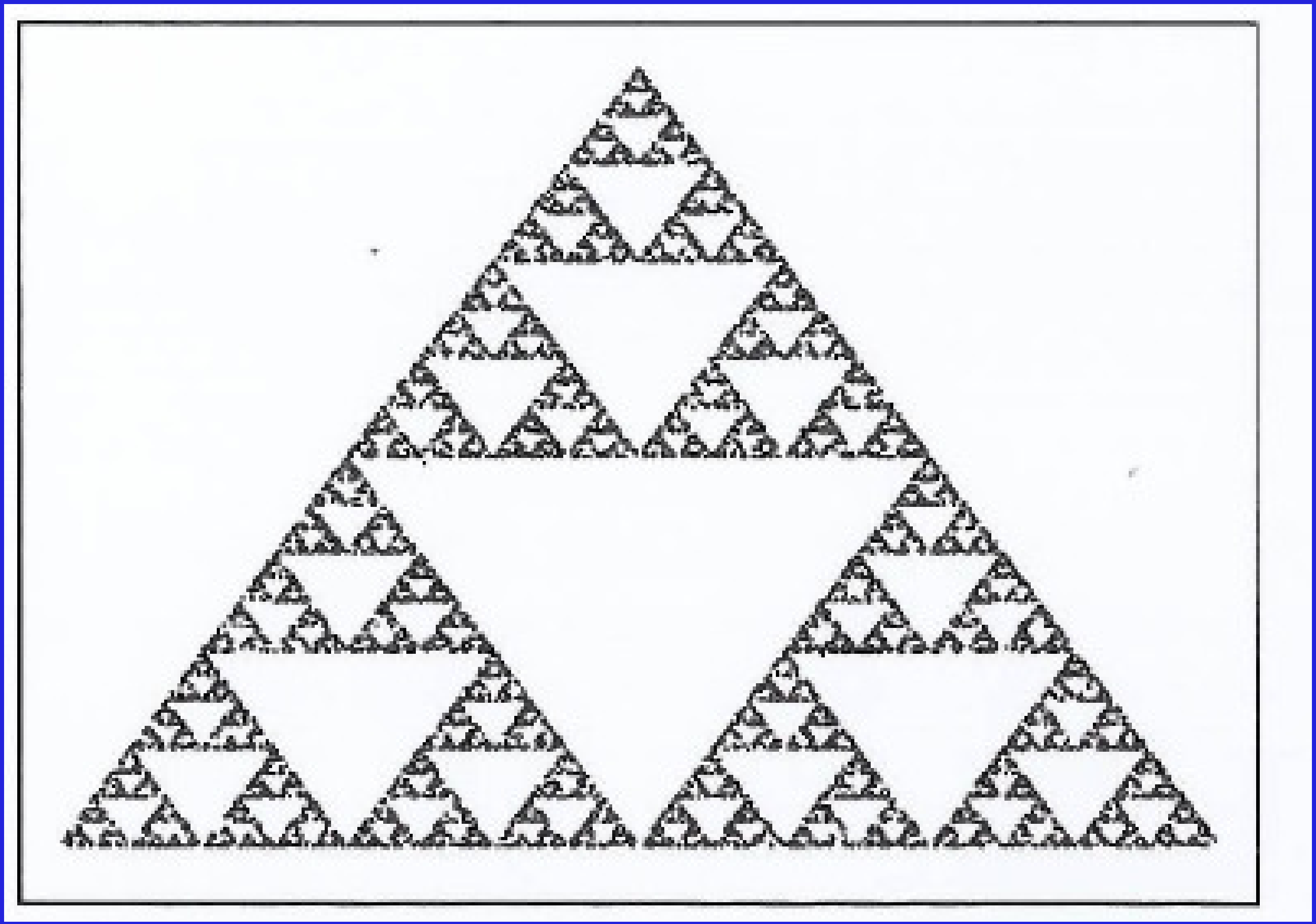
$$\Delta_N = x_N^{(1)} - x_N^{(2)}$$

$$\Delta_N = \epsilon e^{N\lambda}$$

Lyapunov exponent.

Exponential growth in solution difference Δ if λ > 0.

Fractals (Play Time)



Let's try to make it. (Actually not so hard.)

Summary

Chaos ID and Lyapunov exponent

First attempt at making a fractal (Sierpiński's gasket)

Don't suffer in silence. Scream for help!!!