Lecture 20 Review

Fourier series.

Fourier Transforms

Fourier series good for periodic functions.

Fourier “transforms” good for non- periodic functions.

FT → $H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t}\, dt$

Inverse FT → $h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t}\, df$

our choice

FT → $H(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) e^{i\omega t}\, dt$ $\omega = 2\pi f$

Inverse FT → $h(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega) e^{-i\omega f t}\, d\omega$

H(f) is a measure of the contribution of a particular frequency to f (t) .

Fourier Transforms (2)

We can think of ‘time space” and “frequency space.”

h(t) and H(f) have various symmetry properties:

h(t) even	H(f) even
h(t) odd	H(f) odd
h(t) real	H(-f) = [H(f)]*
h(t) imaginary	H(-f) = - [H(f)]*

Parseval’s Theorem:

$$\text{"total power"} \equiv \int_{-\infty}^{\infty} |h(t)|^2 \, dt = \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

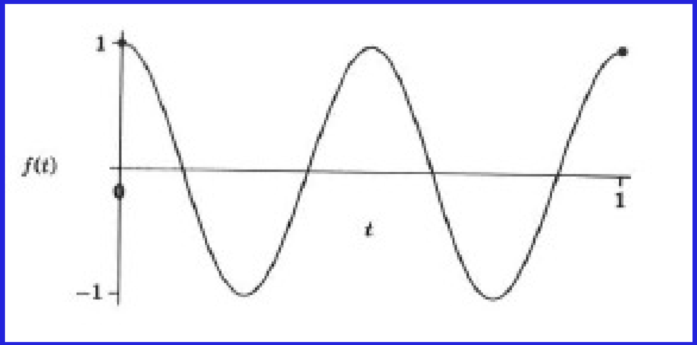
“one-sided power spectral density” (PSD)

$$P_h(f) \equiv |H(f)|^2 + |H(-f)|^2 \quad 0 \leq f < \infty$$

Discrete Fourier Transform

Issue: Often discretely sample a waveform and need to know: its shape and/or its frequency characteristics.

Sample (i.e., measure w/ an instrument) every Δ seconds.
(1/Δ is the "sampling rate")



cos(2πf t)   f = 2
sampled every 1 second

"Nyquist critical frequency" $f_c \equiv \frac{1}{2\Delta}$

Discrete Fourier Transform (2)

Let

$$h_k \equiv h(t_k) \qquad t_k \equiv k\Delta \qquad k = 0, 1, 2 \ldots, N-1$$

$$f_n \equiv \frac{n}{N\Delta} \qquad n = -\frac{N}{2}, \; \ldots \; \frac{N}{2}$$

N = number of data points

Δ = time between data points.

Discretize integral form of FT:

$$H(f_n) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f_n t}\, dt \approx \sum_{k=0}^{N-1} h_k e^{2\pi i f_n t_k} \Delta = \Delta \sum_{k=0}^{N-1} e^{2\pi i k n/N}$$

DFT

$$H_n \equiv \sum_{k=0}^{N-1} h_k e^{2\pi i k n/N}$$

DFT

same units

$$H(f_n) \approx \Delta H_n$$

continuous v. discrete FTs (note the units)

$$h_k = \frac{1}{N} \sum_{n=0}^{N-1} H_n e^{-2\pi i k n/N}$$

Inverse DFT

$$\sum_{k=0}^{N-1} |h_k|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |H_n|^2$$

Discrete form of Parseval's theorem

DFT (3)

$$H_n \equiv \sum_{k=0}^{N-1} h_k e^{2\pi ikn/N}$$

- Note: H_n has periodicity of N. H(n) = H(N-n)*

You do not have 2*N independent numbers in H_n ! Only N.

Same as number of sampled points h_k

(You will see this in DFT data file.)

```
make_fourier_data.cc
dft.cc
inv_dft.cc
```

DFT(4)

All in one place:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{2\pi i f t}\, dt$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{-2\pi i f t}\, df$$

$$H(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} h(t) e^{i\omega t}\, dt$$

$$h(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega) e^{-i\omega f t}\, df$$

DFT

```
make_fourier_data.cc

dft.cc

inv_dft.cc
```

Summary

DFT (theory and lab example).

Don’t suffer in silence. Scream for help!!!