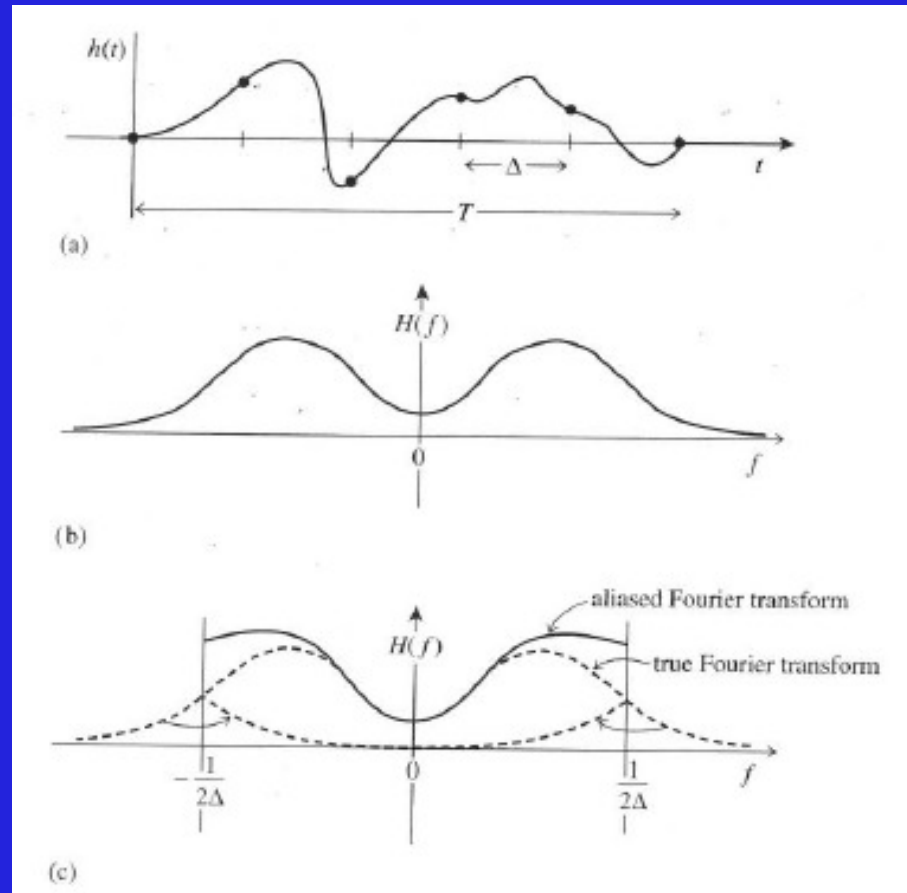


Lecture 22 Review

DFT Review (i.e., significance of H_n).
FFT (+ Nyquist critical frequency)

Sampling Theorem and Aliasing (2)



Limit sampled frequencies to $< f_c$

Fast Fourier Transform (again)

DFT is sloooow. Execution time $\propto N^2$.

Compare $N = 1000$ to $N = 5000$

Fast Fourier Transform (FFT) to the rescue.

Not a new type of transform, but a new way to calculate FT.

$$W \equiv e^{2\pi i/N} \text{ "twiddle factor"}$$

$$H_n \equiv \sum_{k=0}^{N-1} W^{nk} h_k$$

$$F_k = \sum_{j=0}^{N-1} e^{2\pi ijk/N} f_j$$

$$= \sum_{j=0}^{N/2-1} e^{2\pi i(2j)k/N} f_{2j} + \sum_{j=0}^{N/2-1} e^{2\pi i(2j+1)k/N} f_{2j+1}$$

$$= \sum_{j=0}^{N/2-1} e^{2\pi ijk/(N/2)} f_{2j} + W^k \sum_{j=0}^{N/2-1} e^{2\pi ijk/(N/2)} f_{2j+1}$$

repeat...

$$= F_k^e + W^k F_k^o \quad 0 \leq k \leq N-1$$

Execution time $\propto N \log_2 N$

Fast Fourier Transform (2, again)

`make_fourier_data.cc`

`gsl_fft.cc`

`gsl_inv_fft.cc`

GSL FFT routines store $\text{Imag}(H_n)$ differently from DFT
 $H(k) = H(N-k)^*$ (Not all $2N$ $H(k)$ numbers independent.)

- Compare DFT output w/ FFT output.
- Compare execution times. DFT v. FFT w/ $N = 5000$

Octave

Often need to perform calculations quickly (i.e., w/o writing code)

Use octave (freeware)

```
prompt> octave
```

```
octave:1>
```

Use as calculator: $2/83$

Standard set of math functions:

Colon notation:

```
octave:31>: e= 2:6
```

```
octave:31>: f= 2:6:40
```

Semicolon usage:

```
octave:31>: f= 2:6;
```

cos	cosine
exp	exponential
log	Natural log
log10	Lg base 10
tanh	Hyperbolic tangent
atan	Arc-tangent
round	Round to nearest integer

Octave

Matrix manipulation

```
octave:45> a= [ 1, 3; 2, 7]
octave:46> a' ←
octave:47> f = [ 1:6]'
```

Built-in functions for large matrices.

```
octave:51>: s = zeros(M,N) w/ M,N = integers
```

```
octave:52>: r = ones(M,N)
```

```
octave:53>: rr = linspace(x1,x2,N)
```

```
octave:53>: r = logspace(x1,x2,N)
```

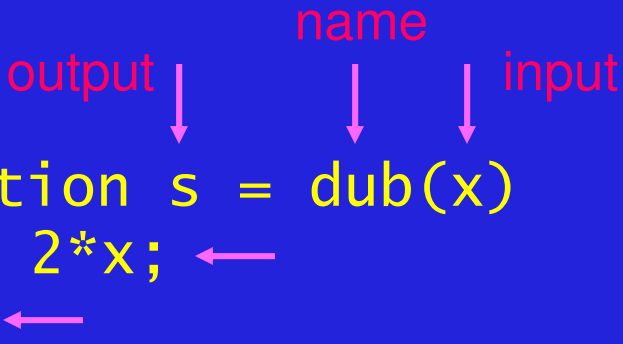
Octave

Plotting: basic command is `plot(x,y)`
uses `gnuplot`

```
octave:85> angles = [ 0:pi/3:2*pi];  
octave:87> y = sin(angles)  
octave:88> plot (angles, y)
```

Functions:

```
octave:151> function s = dub(x)  
    > s = 2*x; ←  
    > end ←  
octave:152> dub(35)
```



Summary

FFT fun.
octave introduction

Don't suffer in silence. Scream for help!!!

