

Lecture 26 Review

Solving ODEs w/ octave (sort of)

Octave 1st ODE

Recall Volterra prey-predator equations

$$\begin{cases} dx/dt = x(a - by) \\ dy/dt = -y(c - dx) \end{cases} \quad a = 1.0, b = 0.5, c = 0.95, d = 0.25$$

$$\frac{dx}{dt} = f(x, t)$$

General form: can be any order ODE

Solved previously w/ GSL routines.

Can be solved w/ octave

```
function xdot = vp(x,t)
```

```
xdot = zeros(2,1);
```

```
a = 1.0;
```

```
b = 0.5;
```

```
c = 0.95;
```

```
d = 0.25;
```

```
xdot(1) = x(1)*(a - b*x(2));
```

```
xdot(2) = -x(2)*(c - d*x(1));
```

```
endfunction
```

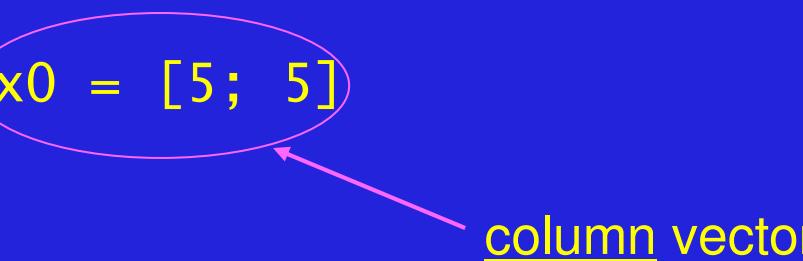
vectors

scalar

How to define a function

REMINDER

1st ODE Solution via octave

- Set initial conditions: $x0 = [5; 5]$

column vector
- $t = \text{linspace}(0, 500, 1000);$

avoids eye strain
- $y = \text{lsode}("vp", x0, t);$

column vector is returned
- $\text{plot}(t, y)$

Q: Can we only solve 1st order ODEs?

If so, this seems (very) limited.

Planetary Orbits via octave

How to solve 2nd order ODEs via octave.

Example: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$

What is trajectory of a body due to F?

$$a_x = \ddot{x} = -\frac{GMx}{(x^2+y^2)^{3/2}}$$

$$a_y = \ddot{y} = -\frac{GMy}{(x^2+y^2)^{3/2}}$$

Recall: $\frac{ds}{dt} = f(s, t)$ octave knows how to solve this ODE

column vector:

math:

$$s \equiv \begin{pmatrix} x \\ v_x \\ y \\ v_y \end{pmatrix}$$

physics:

$$\frac{d}{dt} \begin{pmatrix} x \\ v_x \\ y \\ v_y \end{pmatrix} = \begin{pmatrix} -\frac{v_x}{(x^2+y^2)^{3/2}} \\ -\frac{GMx}{(x^2+y^2)^{3/2}} \\ -\frac{v_y}{(x^2+y^2)^{3/2}} \\ -\frac{GMy}{(x^2+y^2)^{3/2}} \end{pmatrix}$$

2nd order ODE Solutions via octave

kepler.m

```
# function kepler useful for solving  
# planetary motion around sun.
```

```
function xdot = kepler(x,t)  
    xdot = zeros(4,1);  
    a = 39.5;  
    xdot(1) = x(2);  
    xdot(2) = -a*x(1)/((x(1))^2 + (x(3))^2)^1.5;  
    xdot(3) = x(4);  
    xdot(4) = -a*x(3)/((x(1))^2 + (x(3))^2)^1.5;  
  
    endfunction
```

inside octave:

```
source "kepler.m"  
x0 = [1;0;0;2*pi];  
t = linspace(0,2,100);  
y = lsode("kepler", x0, t);  
  
plot(t,y)  
save orbit.dat y
```

orbit.dat from octave

examine contents of orbit.dat

linux[22]> head -10 orbit.dat

inside gnuplot: plot y versus x.

What do you expect?

What do you see?

Return of the van der Pol equation

$$\ddot{x} + \mu(x^2 - a^2)\dot{x} + \omega_0^2x = 0$$

Recall similarity to damped SHM: $\ddot{x} + 2\beta\dot{x} + \omega_0^2x = 0 \quad (\beta > 0)$

Set $a = 1$, $\omega_0 = 1$, $\mu = 0.05$

Plot dx/dt versus x .

What do you expect?

What do you see?

Maxima introduction

“Computer algebra” of great utility.

Incarnated in various canned software packages
(MAPLE, Mathematica, Maxima,)

Maxima will be our choice.

original sw, freeware, Linux/Windows/Mac platform

```
linux/tcsh[25]> xmaxima
```

Starts maxima w/ graphical user interface + tutorial

double-click LMB to execute commands in tutorial.

Maxima introduction (2)

Maxima is powerful. We will use only a small subset of commands.

Begin by playing.

General features: commands end w/ a semicolon (;) or dollar sign (\$)

Quit maxima by typing `quit();`

```
integrate ( $\underbrace{x \cdot \log(x)}_{\text{integrand}}, x);$  variable to integrate over
```

```
diff(%o20,x);  
a:20; (sets a = 20)  
assume (m > 20);  
forget (m > 20);
```

Maxima introduction (3)

See if you can plot solution to van der Pol equation.

Summary

Solving 2bd order ODEs w/ octave

Flirtation with maxima

Don't suffer in silence. Scream for help!!!

