Lecture 26 Review

Solving ODEs w/ `octave` (sort of)

REMINDER

Octave 1st ODE

Recall Volterra prey-predator equations

$$\begin{cases} dx/dt &= x(a-by) \\ dy/dt &= -y(c-dx) \end{cases}$$

a = 1.0, b = 0.5, c = 0.95, d = 0.25

$\frac{dx}{dt} = f(x,t)$ ← General form: can be <u>any</u> order ODE

Solved previously w/ GSL routines.

Can be solved w/ `octave`

vectors

scalar

```
function xdot = vp(x,t)
  xdot = zeros(2,1);
  a = 1.0;
  b = 0.5;
  c = 0.95;
  d = 0.25;
  xdot(1) = x(1)*(a - b* x(2));
  xdot(2) = -x(2)*(c - d*x(1));

endfunction
```

How to define a function

REMINDER

1st ODE Solution via octave

- Set initial conditions: `x0 = [5; 5]`

column vector

- `t = linspace(0, 500, 1000);`
- `y = lsode("vp", x0, t);`

avoids eye strain

column vector is returned

- `plot(t,y)`

Q: Can we only solve 1st order ODEs?

If so, this seems (very) limited.

Planetary Orbits via octave

How to solve 2nd order ODEs via octave.

Example: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$ What is trajectory of a body due to F?

$$a_x = \ddot{x} = -\frac{GMx}{(x^2+y^2)^{3/2}}$$

$$a_y = \ddot{y} = -\frac{GMy}{(x^2+y^2)^{3/2}}$$

Recall: $\frac{ds}{dt} = f(s,t)$ octave knows show to solve this ODE

column vector:

math: $$s \equiv \begin{pmatrix} x \\ v_x \\ y \\ v_y \end{pmatrix}$$

physics: $$\frac{d}{dt}\begin{pmatrix} x \\ v_x \\ y \\ v_y \end{pmatrix} = \begin{pmatrix} v_x \\ -\frac{GMx}{(x^2+y^2)^{3/2}} \\ v_y \\ -\frac{GMy}{(x^2+y^2)^{3/2}} \end{pmatrix}$$

2nd order ODE Solutions via octave

kepler.m

```
# function kepler useful for solving
# planetary motion around sun.

function xdot = kepler(x,t)
    xdot = zeros(4,1);
    a = 39.5;
    xdot(1) = x(2);
xdot(2) = -a*x(1)/((x(1))**2 + (x(3))**2)^1.5;
    xdot(3) = x(4);
xdot(4) = -a*x(3)/((x(1))**2 + (x(3))**2)^1.5;

endfunction
```

inside octave:

```
source "kepler.m"
X0 = [1;0;0;2*pi];
t = linspace(0,2,100);
y = lsode("kepler", x0, t);

plot(t,y)

save orbit.dat y
```

orbit.dat from octave

examine contents of orbit.dat

```
linux[22]> head -10  orbit.dat
```

inside gnuplot: plot y versus x.

What do you expect?

What do you see?

Return of the van der Pol equation

$$\ddot{x} + \mu(x^2 - a^2)\dot{x} + \omega_0^2 x = 0$$

Recall similarity to damped SHM: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$ ($\beta > 0$)

Set a = 1, ω_0 = 1, μ = 0.05

Plot dx/dt versus x.

What do you expect?

What do you see?

Maxima introduction

"Computer algebra" of great utility.

Incarnated in various canned software packages

(MAPLE, Mathematica, Maxima, ….)

Maxima will be our choice.

original sw, freeware, Linux/Windows/Mac platform

linux/tcsh[25]> `xmaxima`

Starts `maxima` w/ graphical user interface + tutorial

double-click LMB to execure commands in tutorial.

Maxima introduction (2)

Maxima is powerful. We will use only a small subset of commands.

Begin by playing.

General features: commands end w/ a semicolon (;) or dollar sign ($)

Quit maxima by typing `quit();`

```
integrate (x*log(x), x);
```

integrand

variable to integrate over

```
diff(%o20,x);
a:20;   (sets a = 20)
assume (m> 20);
forget (m> 20);
```

Maxima introduction (3)

See if you can plot solution to van der Pol equation.

Summary

Solving 2bd order ODEs w/ `octave`

Flirtation with `maxima`

Don’t suffer in silence. Scream for help!!!