Lecture 5 Review

- Basic rule for numerical integration $I = \sum f(x_i)\, w_i$; $i = 1, N$
- Trapezoidal rule: $w_i = \{ h/2, h, h, \ldots h, h, h/2\}$,
  $h = (b - a)/N$, N = odd.
- Simpson's rule: $w_i = \{ h/3, 4h/3, 2h/3, 4h/3, 2h/3, \ldots 2h/3, 4h/3, h/3\}$,
  $h = (b - a)/N$, N = odd.

http://www.physics.smu.edu/devel/coan/3340/simpson.cc

http://www.cplusplus.com/doc/tutorial/control.html

http://www.cplusplus.com/reference/clibrary/cmath/

ERROR ESTIMATION IN NUMERICAL INTEGRATION

WE NEED TO ESTIMATE OUR ERRORS IN OUR NI FORMULAE.

Q: HOW WELL DO WE DO WHEN WE USE MORE POINTS (i.e, INCREASE N)

USE STD RESULTS FROM NUMERICAL METHODS

SEE www.mrbook.com/abramowitz_and_stegun/page_885.htm

$$\int_{x_0}^{x_m} f(x)dx = h\left[\frac{f_0}{2} + f_1 + \ldots f_{m-1} + \frac{f_m}{2}\right] - \frac{m h^3}{12} f''(\xi)$$

(ABS NOTATION) w/ $x_0 < \xi < x_m$

TRAPEZOIDAL RULE: NI ERROR ESTIMATE

$$\text{ERROR} = -\frac{m h^3}{12} \underbrace{f''(\xi)}_{\text{UNKNOWN}}$$

$m = N$

$h \propto 1/N$

$$\Rightarrow \boxed{\text{ERROR} = O(1/N^2) \quad \text{TRAPEZOIDAL RULE}}$$

NB: A&S ~ "EXTENDED" TR

SIMPSON'S RULE:

$$\int_{x_0}^{x_{2n}} f(x)dx = \frac{h}{3}\left[f_0 + 4(f_1 + f_3 + \dots f_{2n-1}) \right. \\ - \frac{nh^5}{90} f^{(4)}(\xi).$$

$n = N/2, \; h \propto 1/N$

SIMPSON'S RULE ERROR ESTIMATE

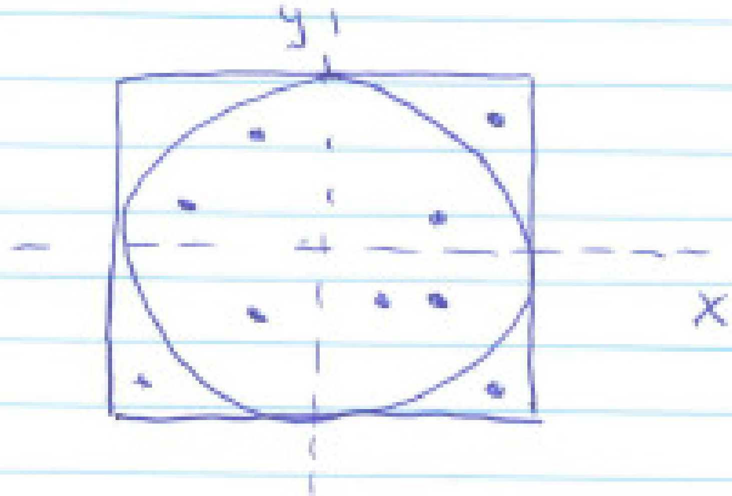
$\int g$ **Error = $O(1/N^4)$**   SIMPSON'S RULE

N.B.: A&S ~ "EXTENDED" SR

http://www.nrbook.com/abramowitz_and_stegun/page_885.htm

MONTE CARLO INTEGRATION

THROW DARTS AT DART BOARD



PROBABILITY OF LANDING IN CIRCLE
PROPORTIONAL TO AREA OF CIRCLE

BUT AREA = INTEGRAL UNDER CURVE

CONSIDER UPPER QUADRANT $0 \le x \le 1$, $0 \le y \le 1$
AREA IN QUAD = $\int y \, dx$

MC INTEGRATION (2)

$$A = \int_0^1 \sqrt{1-x^2}\, dx$$

"THROW A DART" MEANS PICK RANDOM NMBR

RANDOM ≡ CAN'T BE PREDICTED.

PRACTICALLY, DONE BY FUNCTION CALL.

PICK RANDOM X $(0 \le x < 1)$

PICK RANDOM y $(0 \le y \le 1)$

IF $y \le \sqrt{1-x^2}$, THEN DART IN QUADRANT

REPEAT MANY TIMES, SAY, N_T

CALCULATE HOW MANY DARTS -CIRCLE N_Q

$$A = (N_Q/N_T) * \text{AREA OF SQUARE}$$

Random Number Generation

```
// computes some random numbers

#include <iostream>

using namespace std;

int main()
{
  srand((time(0)));  // "seed" the random number generator
  int r = random();

  // RAND_MAX is largest random number. it is an integer. built-in.

  cout << " random number: " << (double) random()/RAND_MAX <<
endl;
  cout << " random number: " << random() << endl;
  cout << " random number: " << random() << endl;

cout << "The value of RAND_MAX is " <<  RAND_MAX << endl;
  return 0;
}
```

Plotting with Gnuplot (just a peek)

We need to be able to plot. Use `gnuplot` utility.

Try it! (Start by typing `gnuplot` to enter utility.)

Usage examples:

```
gnuplot>   plot sin(x)
gnuplot>   plot [1:4] exp(-x) lw 2
gnuplot>   plot “data_file.dat”
```

(data_file.dat: Must already exist)

http://www.gnuplot.info/help.html

http://sparky.rice.edu/~hartigan/gnuplot.html

Link also available from PHYS 3340 links page

Q: plot 500 random numbers (0,1).

Help with NI errors and gnuplot

My head is exploding.

I need something to read quietly, at my own pace.

http://www.nrbook.com/abramowitz_and_stegun/page_885.htm

http://www.gnuplot.info

Link also available from PHYS 3340 links page

Summary

- Errors for trapezoidal & Simpson's NI techniques.
- Monte Carlo ("dart throwing") technique for integration.
- Code for random number generation.
- C++ features: `if`, declaring functions, `random`, ...
- Just a peek at plotting: gnuplot.

http://www.gnuplot.info/help.html

http://sparky.rice.edu/~hartigan/gnuplot.html

Don't suffer in silence. Scream for help!!!