Lecture 7 Review

- Basic `gnuplot` commands
- GSL routine `gsl_rng_uniform` for uniform random number generation in interval (0,1).
- GSL routine `gsl_integration_qags` for NI.

PROBABILITY

$$P = \frac{\text{\# WAYS TO GET "SUCCESS"}}{\text{TTL \# OF WAYS}}$$

PROBABILITY DISTRIBUTION $g(x)$

$P = g(x)dx$

= PROBABILITY OF GETTING A VALUE OF x BETWEEN x & $x + dx$

$$\text{PROB}(a \le x \le b) = \int_a^b g(x)dx$$

NOTE: $1 = \int_{-\infty}^{\infty} g(x)dx$

BINOMIAL PROBABILITY DISTRIBUTION

APPLIES WHEN AN EXPERIMENT HAS ONLY 2 OUTCOMES.

Q: HOW MANY SUCCESSES X AFTER N TRIES, WHEN PROB OF SUCCESS FOR EACH EXPT $\equiv p$?

USE COIN FLIPPING ANALOGY.

FLIP N COINS, X ARE HEADS, $p = 1/2$

BINOMIAL DISTRIBUTION

X HEADS $\qquad$ N−X TAILS

H T H H T ... T H T

$\boxed{H}$ $\qquad\qquad$ $\boxed{T}$

LAY ALL COINS ON TABLE.
PLACE X IN HEADS BOX
PLACE N−X IN TAILS.

HOW MANY WAYS TO PICK THE HEADS?

$$P_m(N,x) = N(N-1)(N-2)\cdots(N-x+2)(N-x+1)$$

"PERMUTATIONS"

$$P_m(N,x) = \frac{N!}{(N-x)!}$$

BINOMIAL DISTRIBUTION

WE ONLY CARE ABOUT THE # OF HEADS, NOT ORDER IN WHICH THEY WERE PICKED.

NEED TO CORRECT. LOOK IN HEADS BOX. HOW MANY WAYS COULD THESE X COINS HAVE ARRIVED?

1ST HEAD COULD HAVE BEEN ANY OF X

2ND H COULD HAVE BEEN ANY OF X-1

SO, ... THERE ARE X! WAYS ALL GIVE SAME # OF HEADS.

$$C(N,X) = \frac{P_m(N,X)}{X!} = \frac{N!}{X!\,(N-X)!} = \binom{N}{X}$$

BINOMIAL DISTRIBUTION

$C(N,x)$ = # OF COMBINATIONS OF PICKING x THINGS FROM A SET OF N THINGS.

$P_B(x; N, p) = C(N,x) * \{$PROB OF HAVING AN ARRANGEMENT OF H's & T's w/ x HEADS AND $N-x$ TAILS$\}$

$= C(N,x)\, P_{HT}$

$P_{HT} = p^x (1-p)^{N-x}$

"HEADS" "TAILS"

$$P_B(x; N, p) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

BINOMIAL DISTRIBUTION

Q: THROW N COINS

w/ SUCCESS PROBABILITY FOR ONE EXPT $= p$

WHAT IS AVERAGE # OF SUCCESSES?

IN GENERAL

$$\langle \text{SOMETHING} \rangle = \int_{-\infty}^{\infty} \text{SOMETHING} \times f(x)\,dx$$

$$\text{MEAN} \quad \mu = \sum_{x=0}^{N} x\, P_B(x; N, p) = Np$$

BINOMIAL DISTRIBUTION

OTHER USEFUL QUANTITY = "VARIANCE"

$$\sigma^2 = \langle (x-\mu)^2 \rangle = \sum_{x=0}^{N} (x-\mu)\, P_B(x; N, p)$$

$$= Np(1-p) = Npq$$

Summary

Binomial probability distribution

$$P_B(x;N,p) = \left(\begin{array}{c} N \\ x \end{array} \right) p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

$\mu = Np$ mean value

$\sigma^2 = Npq$ "variance" σ measures width of P_B.

Don't suffer in silence. Scream for help!!!

Summary

- Binomial probability distribution

$$P_B(x;N,p) = \begin{pmatrix} N \\ x \end{pmatrix} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

- GSL routine `gsl_rng_uniform` for uniform random number generation in interval (0,1).

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