Lecture 8 Review

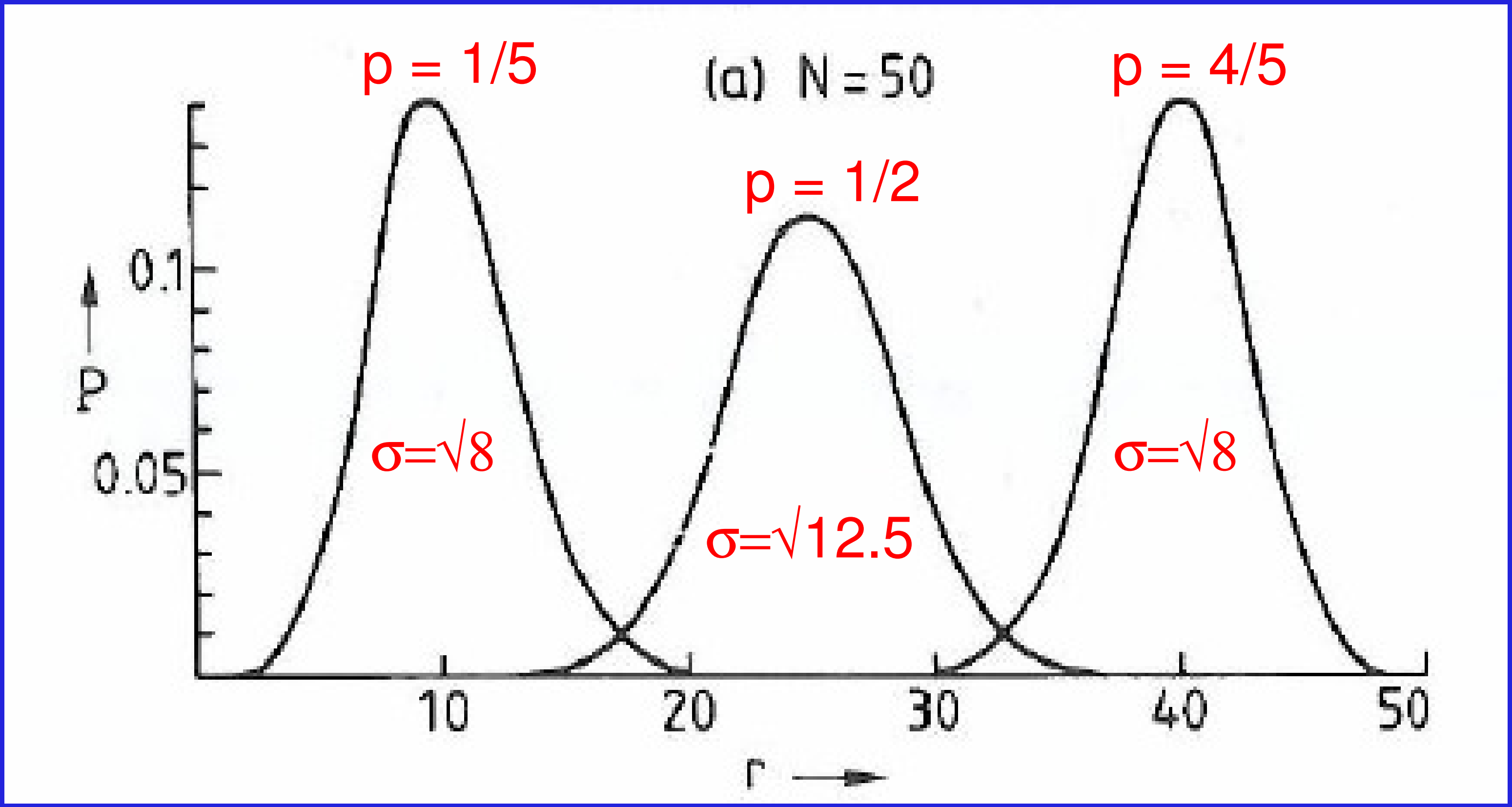
Binomial probability distribution

$$P_B(x; N, p) = \begin{pmatrix} N \\ x \end{pmatrix} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

$\mu = Np$ mean value

$\sigma^2 = Npq$ "variance." σ measures width of P_B.

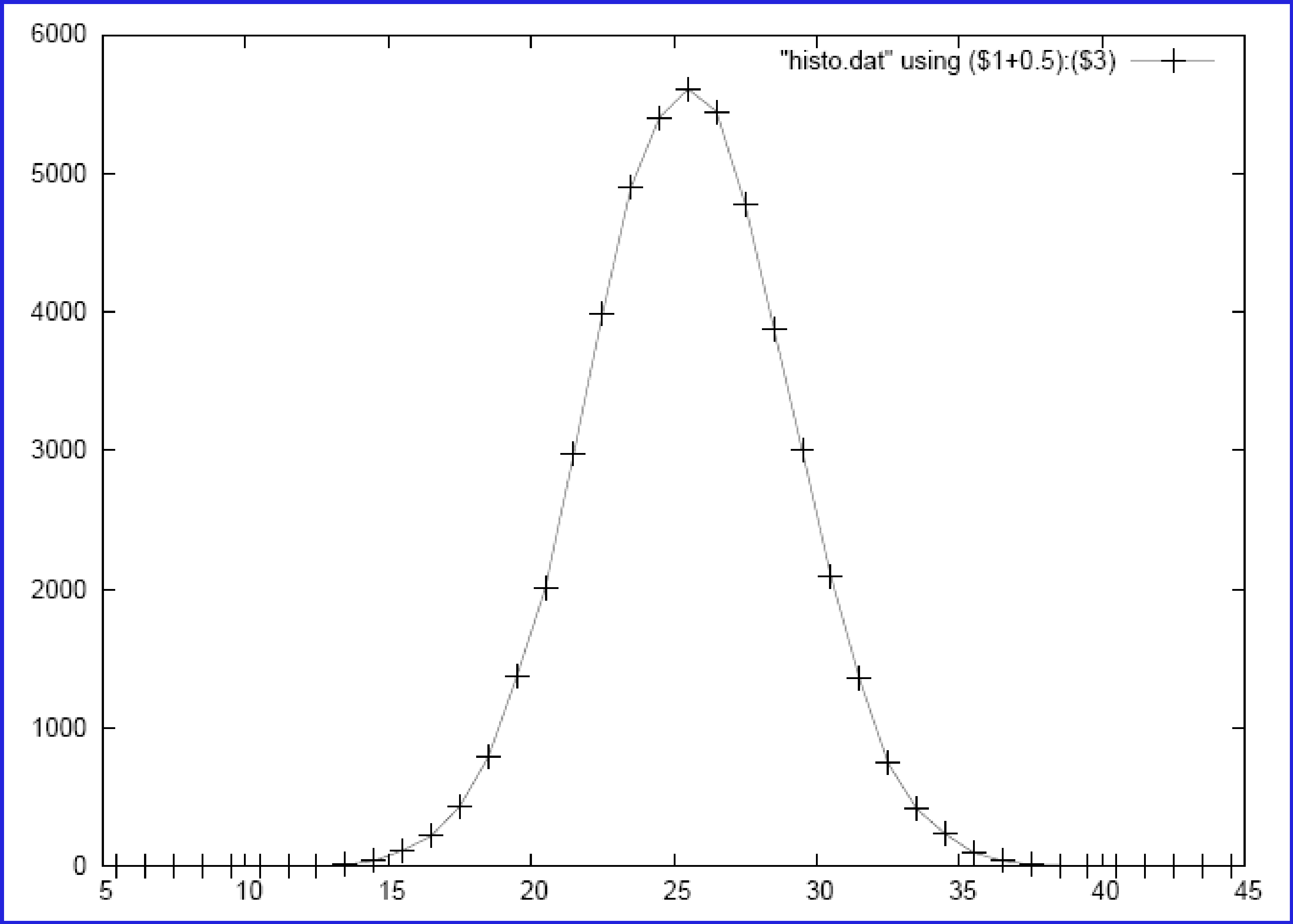
Binomial Distribution Examples



gsl-randist 123 1000 binomial 0.5 50 | gsl-histogram 5 45 40

"pipe" symbol: directs <u>output</u> of one <u>command</u> to input of <u>another</u>.

Binomial Distribution Histogram via gnuplot



gnuplot> `plot “histo.dat” using ($1+0.5):($3) w lp`

Poisson Probability Distribution

- Interesting case when N $\to \infty$, p $\to$ 0, but N*p = finite.

$P_B(x; N, p)$ describes prob of observing x events per unit time

out of N possible, each of which has prob p of occurring.

$$\lim_{p \to 0} P_B(x; N, p) = P_P(x; \mu) \equiv \frac{\mu^x}{x!} e^{-\mu}$$

μ = mean number of events per unit time

$\sigma^2 = \mu$ (σ still characterizes width of probability distribution curve)

Q: Plot poisson distribution w/ μ = 3.5, 10,000 points

Use piping, redirection and `gnuplot`. NO cut-and-paste !!!

Gaussian Probability Density Distribution

- Another interesting case when $N \rightarrow \infty$, $p \neq 0$, but $N^*p \gg 1$

Same case is obtained from $P_P(x; \mu)$ when $\mu \gg 1$

We obtain "gaussian" probability density distribution P_G

$$P_G(x; \sigma, \mu) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

P_G is a continuous function. $P_G\,dx$ is a probability (i.e., a pure number)

$P_G\,dx$ is a probability of observing x between x and x + dx

μ = mean value of x in parent distribution

σ^2 characterizes variation about μ of parent distribution.

Q: Plot gaussian distribution w/ $\mu = 10.0$, $\sigma = 3.0$, 10,000 points

Use piping, redirection and `gnuplot`. NO cut-and-paste !!!

Probability Numbers from Probability Distributions

- Often need to pick a random number

from an arbitrary probability distribution P(x).

How to do this from uniform probability distribution p(r) btwn [0,1)?

$$p(r) = \begin{cases} 1 \text{ for } 0 \leq 1 \\ 0 \text{ otherwise} \end{cases}$$

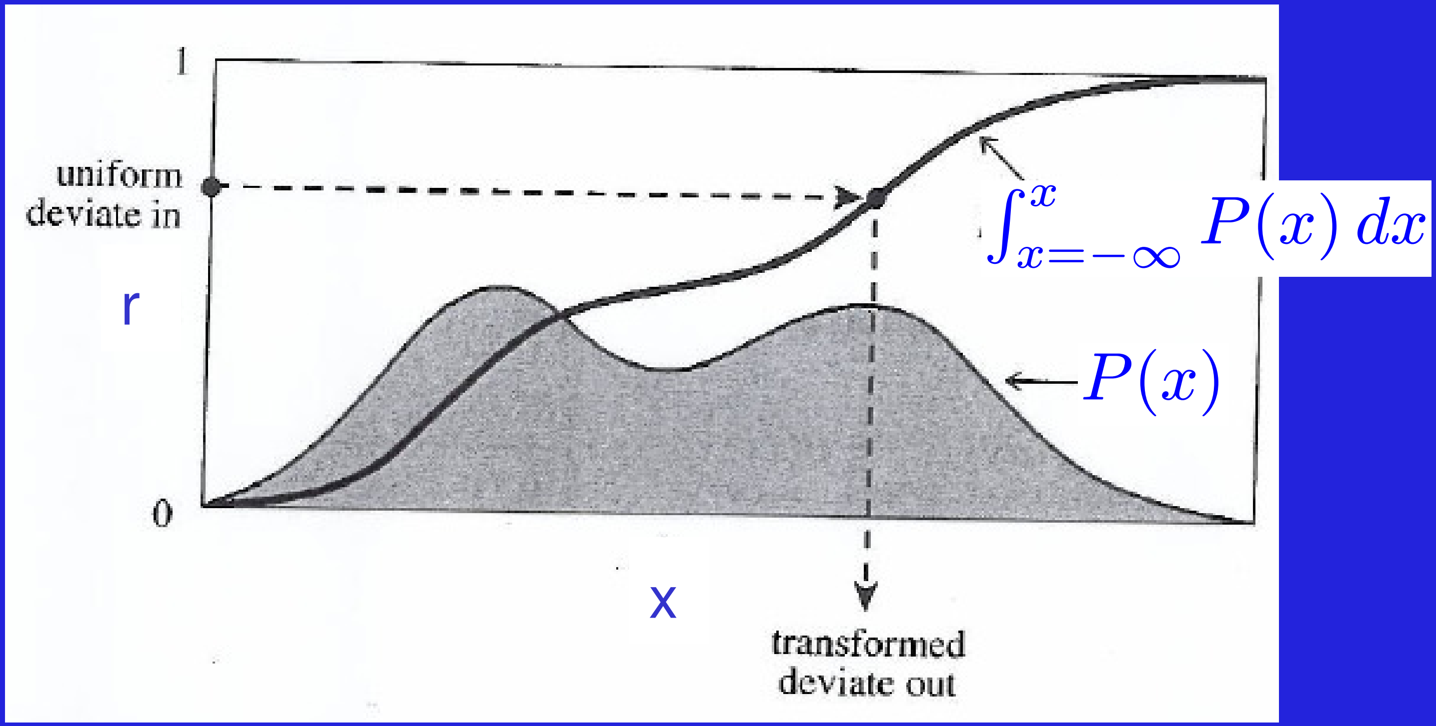
Conservation of probability: $|p(r)\Delta r| = |P(x)\Delta x|$

$$\int_{r=-\infty}^{r} p(r)\, dr = \int_{x=-\infty}^{x} P(x)\, dx$$

$$\int_{r=0}^{r} p(r)\, dr = \int_{x=-\infty}^{x} P(x)\, dx$$

$$r = \int_{x=-\infty}^{x} P(x)\, dx$$

Probability Numbers from Probability Distributions (2)



Fitting Data (first peek)

Need to fit curves to our data (i.e., fit a curve to a histogram)

Do this by minimizing the quantity χ^2

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i} \right)^2$$

y_i is the y value from the curve.

$y(x_i)$ is the y value from the data at a particular x_i.

σ_i is the error on the y(xi). (Use sqrt(nmbr of bin counts) in a histogram.)

Example:

```
gnuplot> f1(x) = a1* sin(b1*x); a1 = 1.0; b1 = 0.3;
gnuplot> fit f1(x) "trash.dat" using ($1):($3) via a1 b1
```

Summary

Poisson and Gaussian probability distributions.

Pick random numbers from arbitrary probability distribution.

First peek at fitting data.

Don’t suffer in silence. Scream for help!!!