Lecture 10 Review

Numerical derivatives: forward and central difference.

ODE solver: 4th order Runge-Kutta
Van der Pol Oscillator Equation

We need an ODE to test drive our RK4 algorithm.

\[ \ddot{x} + \mu \dot{x} (x^2 - 1) + x = 0 \]

“Van der Pol” oscillator equation, \( \mu > 0 \).

Describes a “non-linear” oscillation.
(Restoring force not proportional to displacement)
\( \mu = 0 \Rightarrow \) simple harmonic motion

\[ |x| > 1 \text{ damped oscillatory motion, } x \to 0 \]
\[ |x| < 1 \text{ growing oscillatory motion, } x \to \infty \]

Must be happy medium.

Happy medium seen: plot \( \frac{dx}{dt} \) v. \( x \). “Phase space” diagram.

We will see this w/ rk4.cc and gnuplot.
Rewriting our ODE

\[ \ddot{x} + \mu \dot{x}(x^2 - 1) + x = 0 \]

Recall “standard ‘form” to cast our ODE into. This is a change of variables.

\[ d\vec{y}/dt = \vec{f}(t, \vec{y}) \]

Just a set of first order ODEs. No big deal.

\[ \vec{y} = \begin{pmatrix} y^{(0)}(t) \\ y^{(1)}(t) \\ \vdots \\ y^{(N-1)}(t) \end{pmatrix} \]

\[ \vec{f} = \begin{pmatrix} f^{(0)}(t, \vec{y}) \\ f^{(1)}(t, \vec{y}) \\ \vdots \\ f^{(N-1)}(t, \vec{y}) \end{pmatrix} \]

\[ y^{(0)}(t) = x(t) \quad \text{Displacement of object} \]
\[ y^{(1)}(t) = \dot{x}(t) \quad \text{Velocity of object.} \]
\[ f^{(0)}(t, \vec{y}) = y^{(1)}(t) \]

These 3 guys are always the same for any ODE.
Still Rewriting our ODE

\[ \ddot{x} + \mu \dot{x}(x^2 - 1) + x = 0 \]

We need: \( f^{(1)}(t, \bar{y}) \)

Get this by rearranging ODE and make a simple change of variables:

\[ \ddot{x} = -x - \mu \dot{x}(x^2 - 1) \]
\[ \dot{y}^{(1)} = -y^{(0)} - \mu y^{(1)}((y^{(0)})^2 - 1) \]
\[ \dot{y}^{(1)} = f^{(1)} \]

\[ f^{(1)} = -y^{(0)} - \mu y^{(1)}((y^{(0)})^2 - 1) \]
\[ f^{(0)} = y^{(1)} \]

Using:

\[ y^{(0)}(t) = x(t) \]
\[ y^{(1)}(t) = \dot{x}(t) \]

Initial conditions:

\[ y^{(0)}(0) = x(0) \]
\[ y^{(1)}(0) = \dot{x}(0) \]
/* solves ode via 4th-order Runge-Kutta method */

#include <iostream>
#include <gsl/gsl_errno.h>
#include <gsl/gsl_matrix.h>
#include <gsl/gsl_odeiv.h>

using namespace std;

int main ()
{
    const gsl_odeiv_step_type * T
    = gsl_odeiv_step_rk4;

gsl_odeiv_step * s
    = gsl_odeiv_step_alloc (T, 2);

double mu = 0.05;  // CHANGE ME. damping parameter

    gsl_odeiv_system sys = {func, jac, 2, &mu};

double t = 0.0, t1 = 100.0;   // CHANGE ME. bounds.

double h = 1e-2;          // CHANGE ME. step size

    return GSL_SUCCESS;
}
C++ arrays

(1-dim) Array declaration and initialization:

```cpp
double y[2] = {3.0, 0.0};
```

Type of array elements.

Size of array. Explicit integer value can be omitted. Size is fixed.

Element values.

Arrays start counting at 0 !!! The value of y[0] is 3.0.

Change value of y[0]:

```cpp
y[0] = 45.3;
```

Declare/initialize new variable:

```cpp
double a = y[0];
```

Warning. This is wrong:

```cpp
double b = y[2];
```

Ref: http://www.cplusplus.com/doc/tutorial/arrays.html
C++ while statement

Useful control structure:

```cpp
while (t < t1) {
    Blah;
    Test condition: T or F.
    Various statements.
    Executed if test condition true
    Blah;
    Yada;
}

Note: no ending semicolon (;).
```

→ while Used in rk4.cc ←

Ref: http://www.cplusplus.com/doc/tutorial/control.html
Summary

Van der Pol oscillator & ODE solution via RK4 à la GSL

C++ elements: arrays and while statement

Don’t suffer in silence. Scream for help!!!