Lecture 5 Review

- Basic rule for numerical integration: $\int_a^b f(x)dx = \sum f(x_i)\, w_i \; ; \; i = 1, N$
- Trapezoidal rule: $w_i = \{ h/2, h, h, \ldots h, h, h/2\}$,
  $h = (b - a)/N$, N = odd.
- Simpson's rule: $w_i = \{ h/3, 4h/3, 2h/3, 4h/3, 2h/3, \ldots 2h/3, 4h/3, h/3\}$,
  $h = (b - a)/N$, N = odd.

http://www.physics.smu.edu/devel/coan/3340/simpson.cc

http://www.cplusplus.com/doc/tutorial/control.html

http://www.cplusplus.com/reference/clibrary/cmath/

Error Estimate in Numerical Integration

We need to estimate errors in our NI formulae

Q: How much better do we do when we increase N?

Use standard results from numerical analysis.

For trapezoidal rule (what A &S call “extended” trapezoidal rule

See www.nrbook.com/abramowitz_and_stegun/page_885.htm

$$\int_{x_0}^{x_m} f(x)dx = h[\frac{f_0}{2} + f_1 + \cdots + f_{m-1} + \frac{f_m}{2}] - \frac{mh^3}{12} f''(\xi)$$

w/ $x_0 < \xi < x_m$

Trapezoidal Rule: NI Error Estimate

Integration error ~ $-\frac{mh^3}{12} f''(\xi)$

unknown

In our notation:

m = N

h ∝ 1/N

Error ~ $O(1/N^2)$

Trapezoidal Rule

N.B.: A & S “extended” TR

Simpson's Rule & Error Estimate

$$\begin{aligned}\int_{x_0}^{x_{2n}} f(x)dx = & \quad \frac{h}{3}[f_0 \\ & + \quad 4(f_1 + f_3 + \ldots f_{2n-1}) \\ & + \quad 2(f_2 + f_4 + \ldots + f_{2n-2}) \\ & + \quad f_{2n}] \\ & - \quad \frac{nh^5}{90} f^{(4)}(\xi)\end{aligned}$$

n = N/2

h ∝ 1/N

Error ~ $\mathcal{O}(1/N^4)$

N.B.: A & S "Extended" SR

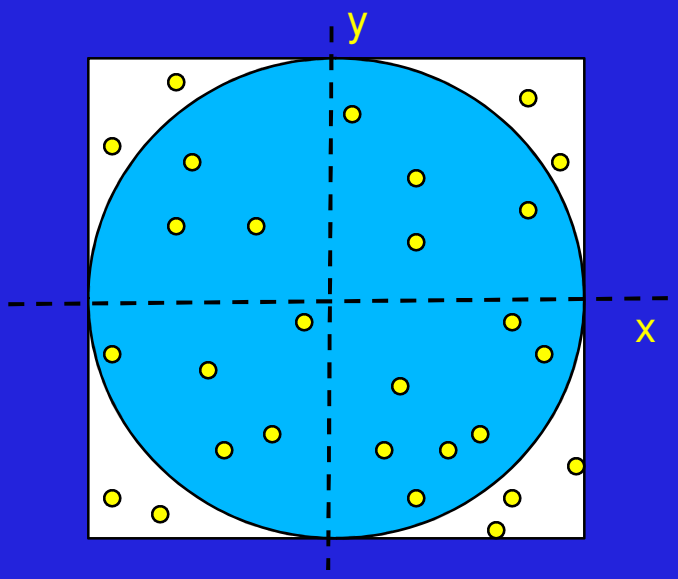
∴ Much better than trapezoidal rule

More accurate for same number of points N

http://www.nrbook.com/abramowitz_and_stegun/page_885.htm

Monte Carlo Integration

Throw darts at a dart board



Probability of landing in the circle
Proportional to area of circle

BUT, area = integral under curve
Consider now, upper quadrant: $0 \le x \le 1$, $0 \le y \le 1$
Area of upper quadrant = $\int y\, dx$

$$A = \int_0^1 \sqrt{1 - x^2}$$

Monte Carlo Integration (2)

$$A = \int_0^1 \sqrt{1 - x^2}$$

"throw a dart" means pick a "random" number

random = can't be predicted

Practically speaking, done by a function call

Pick random x, $0 \le x \le 1$

Pick random y, $0 \le y \le 1$

IF $y \le \sqrt{1 - x^2}$ THEN, dart is in quadrant

means under the curve

Repeat process many times, say, N_T

Count how many darts in quadrant, call it, N_U

A = (N_U / N_T) * area of square

Random Number Generation

```
// computes some random numbers

#include <iostream>

using namespace std;

int main()
{
  srand((time(0)));  // "seed" the random number generator
  int r = random();

  // RAND_MAX is largest random number. it is an integer. built-in.

  cout << " random number: " << (double) random()/RAND_MAX <<
endl;
  cout << " random number: " << random() << endl;
  cout << " random number: " << random() << endl;

cout << "The value of RAND_MAX is " <<  RAND_MAX << endl;
  return 0;
}
```

Plotting with Gnuplot (just a peek)

We need to be able to plot. Use `gnuplot` utility.

Try it! (Start by typing `gnuplot` to enter utility.)

Usage examples:

```
gnuplot>   plot sin(x)
gnuplot>   plot [1:4] exp(-x) lw 2
gnuplot>   plot "data_file.dat"
```

Must already exist

http://www.gnuplot.info/help.html

http://sparky.rice.edu/~hartigan/gnuplot.html

Links also available from PHYS 3340 links page

Q: plot 500 random numbers (0,1).

Help with NI errors and gnuplot

My head is exploding.

I need something to read quietly, at my own pace.

http://www.nrbook.com/abramowitz_and_stegun/page_885.htm

http://www.gnuplot.info

Link also available from PHYS 3340 links page

Summary

- Errors for trapezoidal & Simpson's NI techniques.
- Monte Carlo ("dart throwing") technique for integration.
- Code for random number generation.
- C++ features: `if`, declaring functions, `random`, …
- Just a peek at plotting: gnuplot.

http://www.gnuplot.info/help.html

http://sparky.rice.edu/~hartigan/gnuplot.html

Don't suffer in silence. Scream for help!!!