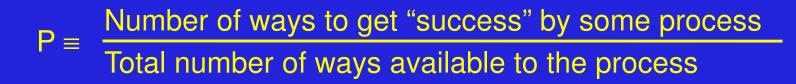
Lecture 7 Review

Basic gnuplot commands

• GSL routine gs1_rng_uniform for uniform random number generation in interval [0,1).

• GSL routine gs1_integration_qags for NI.

Probability



NOTE: $0 \le P \le 1$

P= 0: success <u>never</u> occurs P= 1: success <u>always</u> occurs

Probability "density" g(x)

 $\mathsf{P}(\mathsf{x}) = \mathsf{g}(\mathsf{x})\mathsf{d}\mathsf{x}$

= probability of getting a value of x btwn x & x + dx

NOTE: P(x) need not be constant.

jargon: P(x) = probability "distribution"

$$P(a \le x \le b) = \int_a^b g(x) dx$$

 $1 = \int_{-\infty}^{\infty} g(x) dx$

"something must happen"

Binomial Probability Distribution

Relevant when an experiment has ONLY 2 outcomes each time you run it.

Q: How many successes x after N tries, If, probability of success for <u>each</u> try is p?

Example

Use coin flipping analogy

Flip N coins, x = number of heads (our "successes"), $p = \frac{1}{2}$.

Binomial Distribution

Flip N coins: x heads N - x tails HTTHH....THTTH Lay all coins on a table in a row, say. Place x of them in the "heads" box Place N-x of them in the "tails" box H T

Q: How many unique ways to flip a coin N times AND end up w/ x heads in the heads box?

Q1: How many <u>unique</u> ways to flip N times and achieve x heads?

$$P_M(N,x) = N(N-1)(N-2)\dots(N-x+2)(N-x+1)$$

"permutations"

$$P_m(N,x) = \frac{N!}{(N-x)!}$$

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Binomial Distribution (2)

- · We only care about total number of heads, not the order of appearance
- Our formula for $P_m(N,x)$ would over count if we used it as is.
- Need to correct by ignoring <u>order</u> of the heads.
- Look in heads box. How many ways could these x coins have arrived?

1st head could have been <u>any</u> of the x heads.2nd head could have been <u>any</u> of the remaining x-1 heads.3rd head could have been <u>any</u> of the remaining x-2 heads.

SO There could have been x! ways of arriving from the table to the box. Each way ends up w/ x heads in the heads box.

$$C(N,x)=rac{P_m(N,x)}{x!}=rac{N!}{(N-x)!x!}=\left(egin{array}{c}N\\x\end{array}
ight)$$

The answer to our boxed question.

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Binomial Distribution (3)

C(N,x) = # of <u>combinations</u> of picking x things from a set of N things Relevant when the <u>order</u> of the x things is irrelevant.

 $P_B(x; N,p) = C(N,x) * \{Probability of having an arrangement of H's & T's w/ x heads and (N-x) tails.\}$

 $= C(N,x) * P_{HT}$ $P_{HT} = p^{x}(1-p)^{N-x}$ $P_{HT} = p^{x}(1-p)^{N-x}$ $P_{HT} = p^{x}(1-p)^{N-x}$ $P_{HT} = \frac{N!}{(N-x)!x!}p^{x}(1-p)^{N-x}$

p = probability of a success on an individual flip

Binomial Distribution (4)

Flip N coins w/ probability of success p for each flip. Q: What is the mean number of successes?

In general, for any probability density,

 $\langle Something \rangle = \int_{-\infty}^{\infty} Something \times g(x) \, dx$

For the case of the binomial probability distribution,

mean x value $\mu \equiv$

$$\equiv \langle x \rangle = \sum_{x=0}^{N} x P_B(x; N, p) = Np$$

Npq

Other useful quantity, "variance." Measure of spread of x values around μ .

$$\sigma^2 \equiv \langle (x-\mu)^2 \rangle = \sum_{x=0}^N (x-\mu)^2 P_B(x;N,p) = Np(1-p)$$

"standard deviation" = σ

Summary

Binomial probability distribution

$$P_B(x; N, p) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$





Don't suffer in silence. Scream for help!!!

