Lecture 7 Review

- Basic `gnuplot` commands
- GSL routine `gsl_rng_uniform` for uniform random number generation in interval [0,1).
- GSL routine `gsl_integration_qags` for NI.

Probability

$$P \equiv \frac{\text{Number of ways to get "success" by some process}}{\text{Total number of ways available to the process}}$$

$\boxed{\text{NOTE: } 0 \le P \le 1}$

P= 0: success <u>never</u> occurs
P= 1: success <u>always</u> occurs

Probability "density" g(x)

P(x) = g(x)dx

= probability of getting a value of x btwn x & x + dx

$\boxed{\text{NOTE: P(x) need not be constant.}}$

jargon: P(x) = probability "distribution"

$$P(a \le x \le b) = \int_a^b g(x)dx$$

$$1 = \int_{-\infty}^{\infty} g(x)dx$$

"something must happen"

Binomial Probability Distribution

Relevant when an experiment has ONLY 2 outcomes each time you run it.

Q: How many successes x after N tries,
If, probability of success for <u>each</u> try is p?

Example

Use coin flipping analogy

Flip N coins, x = number of heads (our "successes"), p = ½.

Binomial Distribution

Flip N coins: x heads N – x tails

H T T H H ....T H T T H

Lay all coins on a table in a row, say.

Place x of them in the "heads" box

Place N-x of them in the "tails" box

H T

Q: How many unique ways to flip a coin N times AND end up w/ x heads in the heads box?

Q1: How many unique ways to flip N times and achieve x heads?

$$P_M(N,x) = N(N-1)(N-2)\ldots(N-x+2)(N-x+1)$$

"permutations"

$$P_m(N,x) = \frac{N!}{(N-x)!}$$

Binomial Distribution (2)

- We only care about **total** number of heads, not the order of appearance
- Our formula for $P_m(N,x)$ would over count if we used it as is.
- Need to correct by ignoring order of the heads.

➢ Look in heads box. How many ways could these x coins have arrived?

1st head could have been any of the x heads.

2nd head could have been any of the remaining x-1 heads.

3rd head could have been any of the remaining x-2 heads.

SO …. There could have been x! ways of arriving from the table to the box.

Each way ends up w/ x heads in the heads box.

$$C(N,x) = \frac{P_m(N,x)}{x!} = \frac{N!}{(N-x)!x!} = \binom{N}{x}$$

The answer to our boxed question.

Binomial Distribution (3)

C(N,x) = # of <u>combinations</u> of picking x things from a set of N things

Relevant when the <u>order</u> of the x things is irrelevant.

P_B(x; N,p) = C(N,x) $*$ {Probability of having an arrangement of H's & T's w/ x heads and (N-x) tails.}

= C(N,x) $*$ P_{HT}

$$P_{HT} = \underbrace{p^x}_{\text{“heads”}} \underbrace{(1-p)^{N-x}}_{\text{“tails”}}$$

p = probability of a success on an individual flip

$$P_B(x;N,p) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{(N-x)!x!} p^x (1-p)^{N-x}$$

Binomial Distribution (4)

Flip N coins w/ probability of success p for each flip.

Q: What is the mean number of successes?

In general, for any probability density,

$$\langle Something \rangle = \int_{-\infty}^{\infty} Something \times g(x)\, dx$$

For the case of the binomial probability distribution,

mean x value $\mu \equiv \langle x \rangle = \sum_{x=0}^{N} x P_B(x; N, p) = Np$

Other useful quantity, "variance."
Measure of spread of x values around μ.

$$\sigma^2 \equiv \langle (x-\mu)^2 \rangle = \sum_{x=0}^{N} (x-\mu)^2 P_B(x; N, p) = Np(1-p)$$

$$= Npq$$

"standard deviation" = σ

Summary

Binomial probability distribution

$$P_B(x;N,p) = \left(\begin{array}{c} N \\ x \end{array} \right) p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

$\mu = Np$ mean value

$\sigma^2 = Npq$ “variance” σ measures width of P_B.

Don’t suffer in silence. Scream for help!!!