Lecture 8 Review

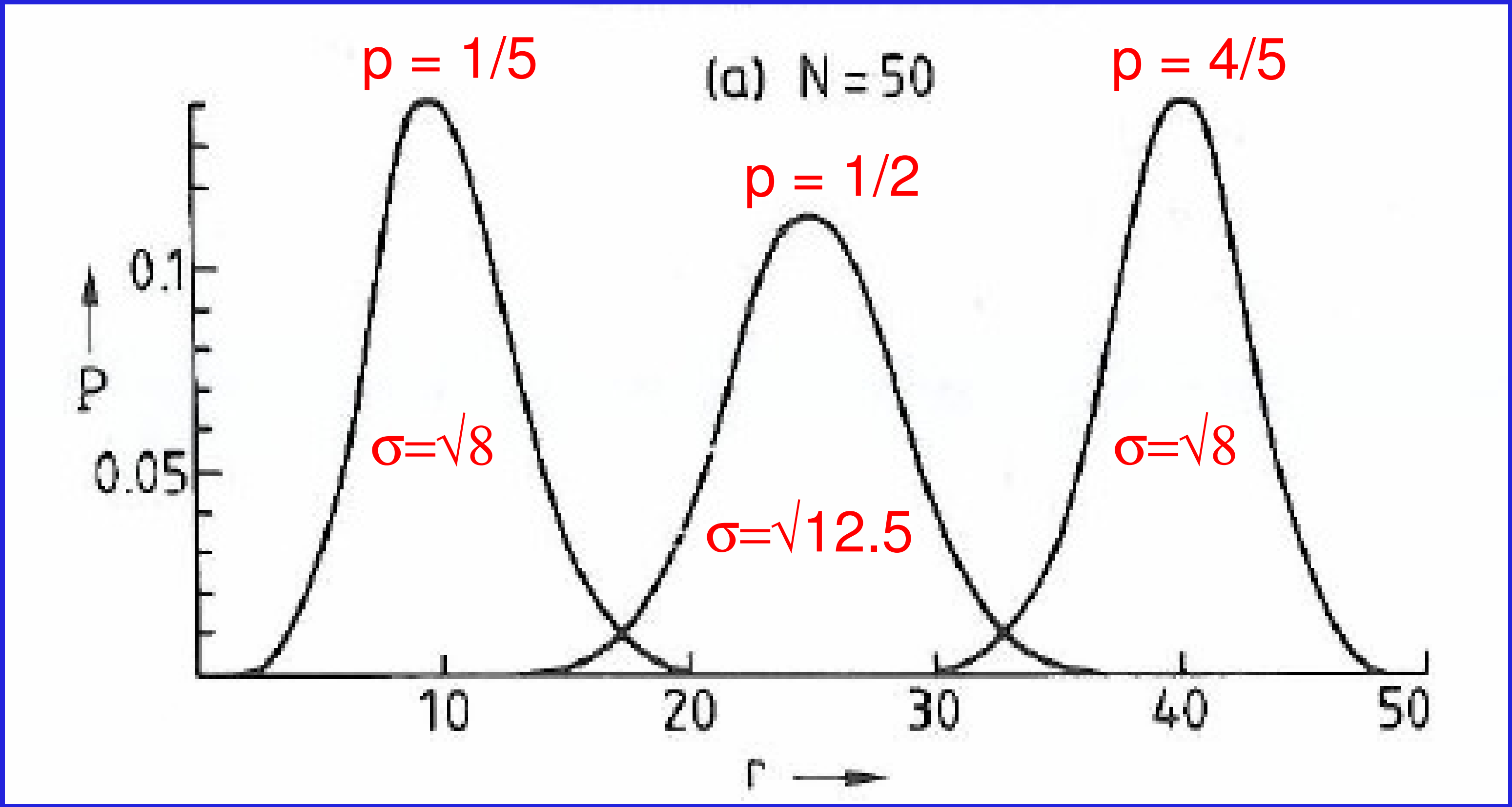
Binomial probability distribution

$$P_B(x; N, p) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

$\mu = Np$ mean value

$\sigma^2 = Npq$ "variance." σ measures width of P_B.

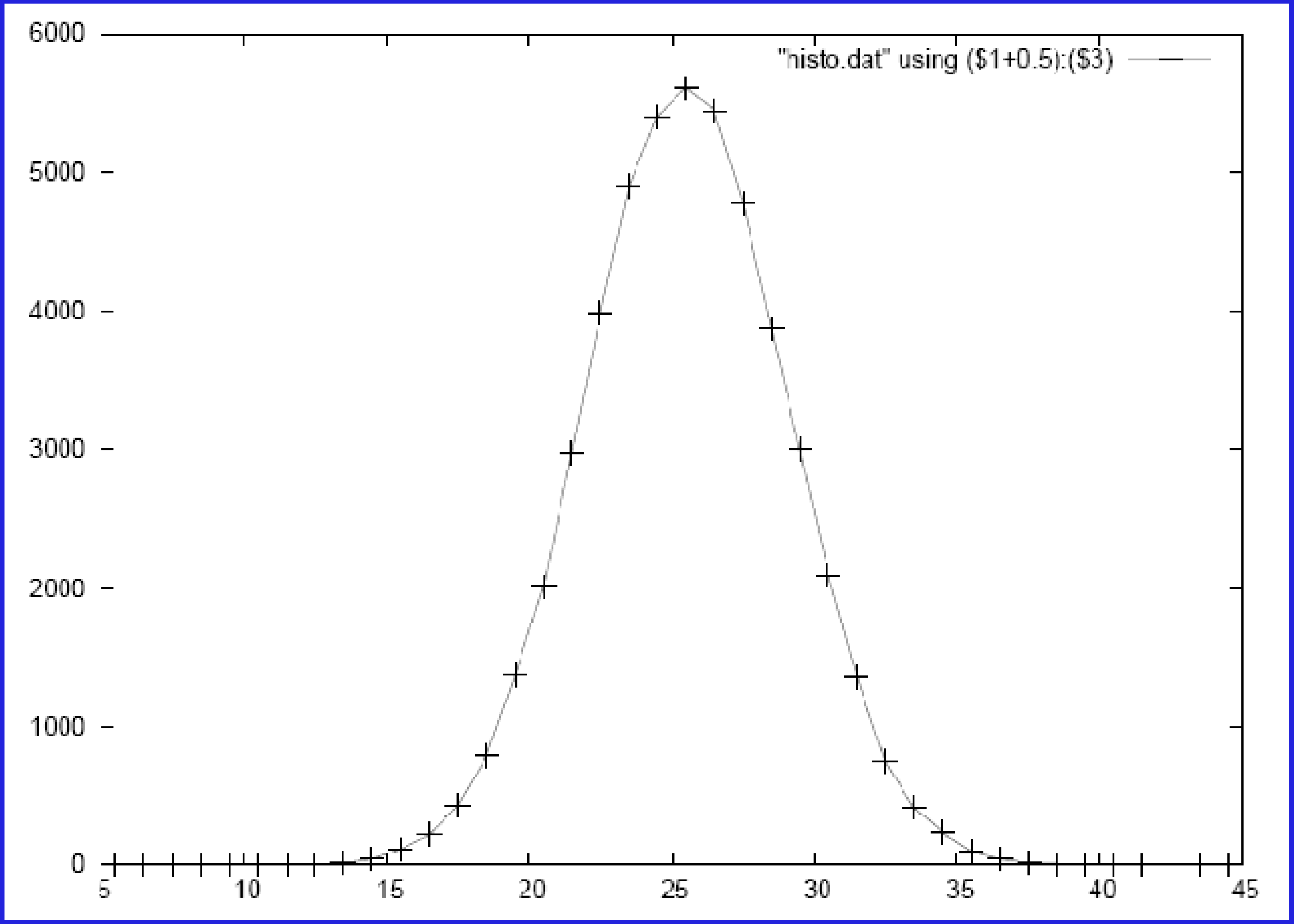
Binomial Distribution Examples



gsl-randist 123 1000 binomial 0.5 50 | gsl-histogram 5 45 40

"pipe" symbol: directs <u>output</u> of one command to <u>input</u> of another.

Binomial Distribution Histogram via gnuplot



gnuplot> `plot “histo.dat” using ($1+0.5):($3) w lp`

Poisson Probability Distribution

- Interesting case when N $\to \infty$, p $\to$ 0, but N*p = finite.

  $P_B(x; N, p)$ describes prob of observing x events per unit time out of N possible, each of which has prob p of occurring.

  Taking the limits,

$$\boxed{\lim_{p\to 0} P_B(x;N,p) = P_P(x;\mu) \equiv \frac{\mu^x}{x!} e^{-\mu}}$$

$P_P(x{:}\ \mu)$ is the probability of observing x occurrences per unit measure when the mean number of occurrences per unit measure is μ.

μ = mean number of events per unit time or per unit length or per unit …

$\sigma^2 = \mu$ (σ still characterizes width of probability distribution curve)

Q: Plot poisson distribution w/ μ = 3.5, 10,000 points
Use piping, redirection and `gnuplot`. NO cut-and-paste !!!

Gaussian Probability Density

- Another interesting case when N $\rightarrow \infty$, p $\neq$ 0, but N*p $\gg$1

  Same case is obtained from $P_P(x; \mu)$ when $\mu \gg$ 1

  We obtain "gaussian" probability density p_G

$$p_G(x;\sigma,\mu) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

p_G is a continuous function. p_G dx is a probability (i.e., a pure number)

p_G dx is a probability of observing x between x and x + dx

μ = mean value of x in parent distribution

σ^2 characterizes variation about μ of parent distribution.

Q: Plot gaussian distribution w/ μ = 10.0, σ = 3.0, 10,000 points

Use piping, redirection and `gnuplot`. NO cut-and-paste !!!

Note: gsl gaussian takes only sigma (assumes μ=0)

Probability Numbers from Probability Distributions

- Often need to pick a random number from an arbitrary probability distribution P(x).

  How to do this from <u>uniform</u> probability distribution p(r) btwn [0,1)?

$$p(r) = \begin{cases} 1 \text{ for } 0 \leq 1 \\ 0 \text{ otherwise} \end{cases}$$

Consider the transformation x(r): random x as a function of uniform random r

The probability distribution of x, g(x)dx, is determined by the condition:

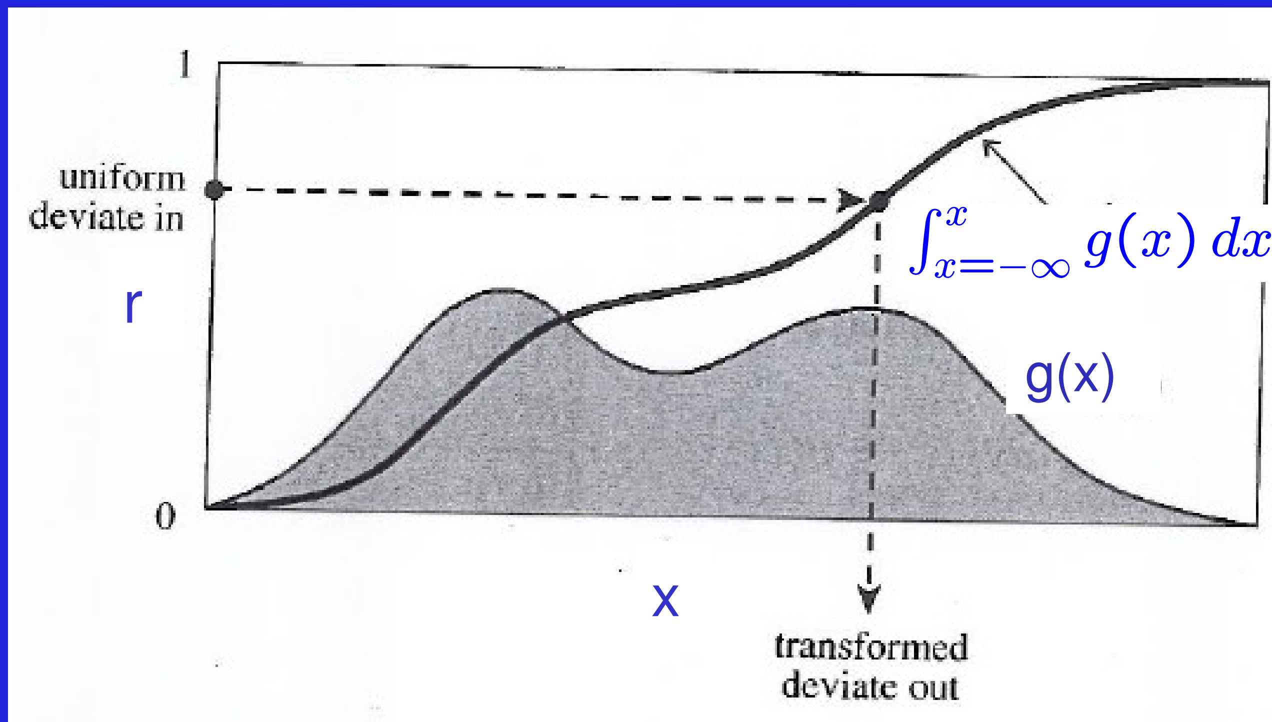
Conservation of probability: $|p(r)\Delta r| = |g(x)\Delta x|$

$$\int_{r=-\infty}^{r} p(r)\, dr = \int_{x=-\infty}^{x} g(x)\, dx$$

$$\int_{r=0}^{r} p(r)\, dr = \int_{x=-\infty}^{x} g(x)\, dx$$

$$r = \int_{x=-\infty}^{x} g(x)\, dx$$

Probability Numbers from Probability Distributions (2)



Fitting Data (first peek)

Need to fit curves to our data (i.e., fit a curve to a histogram)

Do this by minimizing the quantity χ^2

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i} \right)^2$$

y_i is the y value from the curve.

$y(x_i)$ is the y value from the data at a particular x_i.

σ_i is the error on the y(xi). (Use sqrt(nmbr of bin counts) in a histogram.)

Example:

```
gnuplot> f1(x) = a1* sin(b1*x); a1 = 1.0; b1 = 0.3;
gnuplot> fit f1(x) “trash.dat” using ($1):($3) via a1, b1
```

Summary

Poisson and Gaussian probability distributions.

Pick random numbers from arbitrary probability distribution.

First peek at fitting data.

Don’t suffer in silence. Scream for help!!!