Relativity & Gravitation

Newtonian Mechanics L7p1

- Depends completely on the notion of universal time.

  - The Forces that the sun & planets exert on each other are determined by respective distances from each other at the same time.

  - "Action at a distance" is at the heart of Newtonian Gravity.

But in Relativity,

- There is a relative space & time which depends on the observer

  As a result, ambiguities arise

Consider also

$$F_i=m_i a$$

$$F_g=\frac{-G\, m_g\, M}{r^2} = m_g\, g$$

Where "g" is constant gravitational acceleration near the surface of the earth ($\approx 9.80\ m/s^2$)
Then $m_i=m_g$ for all m_i.

General Relativity L7p2

- Mass has two different meanings,
  - Gravitational property
    - Indicates strength with which Gravitational force acts on a body by another
  - Inertial property
    - Represents a resistance to acceleration
    - Value of G in $F_g=\frac{-G\, m_1\, m_2}{r^2}$

      is chosen so that $m_{grav}= m_{inertial}$

The ratio of $m_{grav}/m_{inertial}$ is the same over 10^{12} orders of magnitude in mass.

Why?

Consider:

1. you are in elevator in otherwise empty space accelerated upward by "g"=9.8 m/s^2.
2. You are standing on the surface of Earth

You can't distinguish 1) from 2). Therefore they are equivalent.

Free Fall L7p3

In the Special Theory of relativity: Go to a frame where there are no accelerations.

- $\therefore$ far from other masses (i.e. Gravity)

Acceleration is seems special because we can feel it. Is this true?

So, how do we detect gravity?

- For solar gravity, look at acceleration of all planets toward sun.
- But m_i & m_g are equivalent & feel the same acceleration → so look at acceleration differences.
- Examine a point mass at the same position as an object, there are no acceleration differences between the point mass & the object.
    - Think of the frame of reference (local)
    - Acceleration due do gravity is undetectable by a frame that is accelerated in time with physical objects subject to gravity & no other forces.

"free fall reference frame": cannot, however, be extended arbitrarily far thru space & time since the strength of gravity will change.

Free Falling Reference Frame L7p4

- "With respect to a free-falling frame of reference, material bodies will be unaccelerated if they are free from non-gravitational forces."
- This is the same as inertial frames of reference when there is no gravity present
    - When there is NO gravity present, a frame of reference is extensible (can be extended in space)
- But when gravity is present, we only can construct a local frame of reference

    $\therefore$ the presence of gravity is tantamount to the non-extensibility of local free falling frames of reference

    - The study of gravity is replaced with the study of inhomogeneities.

Postulates of General Relativity L7p5

All local frames of reference are equally valid – there is no way to choose a class of preferred frames of reference with which to formulate the laws of nature.

<u>Principle of Covariance</u>

All laws of nature have the same form for observers in any frame of reference, whether that frame is accelerated or not.

<u>Principle of Equivalence</u>

"In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects."

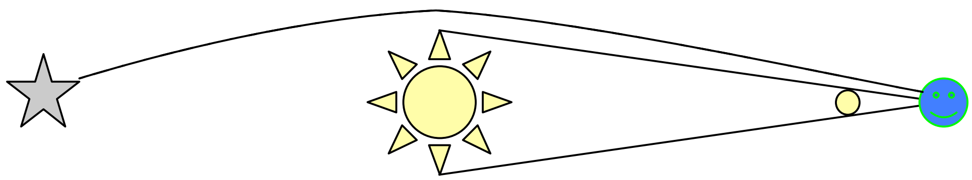
Bending of Light   L7p6

To extend an inertial frame through space & time

- Must compare distant clocks & straight rulers
    - Requires light to carry information

But light has energy, & by Special Relativity, it is subject to gravity

- Light cannot exceed or fall short of “c”
    - But light can change direction

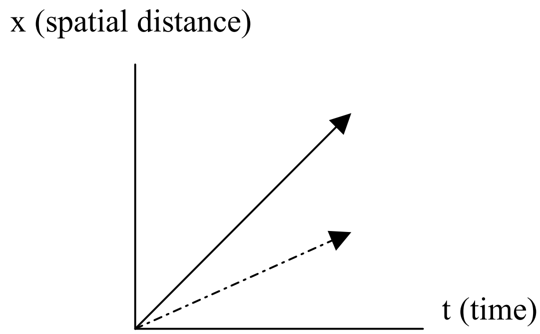


Solar Eclipse 1919 – Starlight observed bent by gravity of the sun!

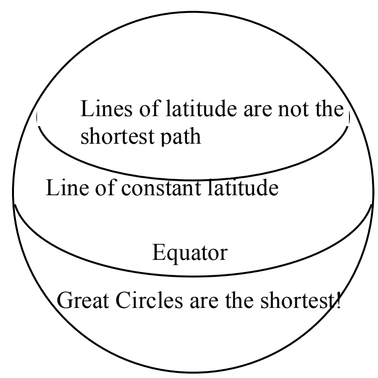
Spacetime L8p1

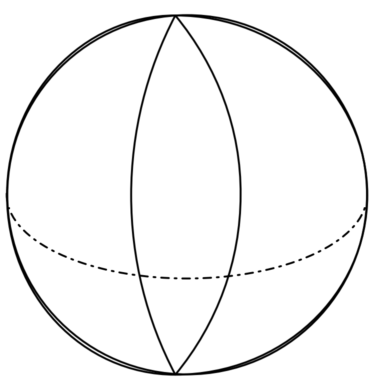
- 4 dimensional geometry : x, y, z, <u>time</u>,
  - o Frame of reference is a 4D coordinate system

  - o Light always takes the "quickest" path between any two points in spacetime.
    - ▪ Shortest distance in spacetime a "geodesic" (a straight line on a curved surface or in a curved space).



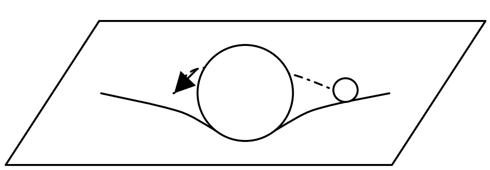
  - o Think of a ball
    (consider 2 people walking North starting at the equator;
    their paths will converge.
  - o Since light's path is bent
    <u>Spacetime itself is curved</u>!





Relativistic Gravity L8p2

It turns out the geometry of curved spacetime can have many possibilities
How can we figure out possibility is true?



Geometry affects motion
- o Acceleration in this case

- We need to find geometry which presents accelerations we see

- The combination of the Principle of Covariance & Newton's Gravitational Law gives a unique Theory of General Relativity.
- Gravity is interpreted as a curvature of a 4D spacetime geometry
  "Acceleration of particles relative to a free-falling reference frame corresponds to a deviation from a geodesic curve. From a choice of coordinate axes

Geometry & Cosmological Constant L8p3

Closed (positively curved), Open (negatively curved) & Flat (zero curvature, also "Open")

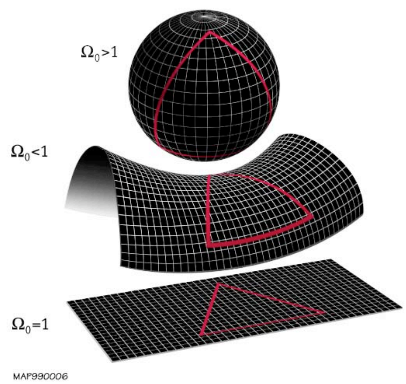
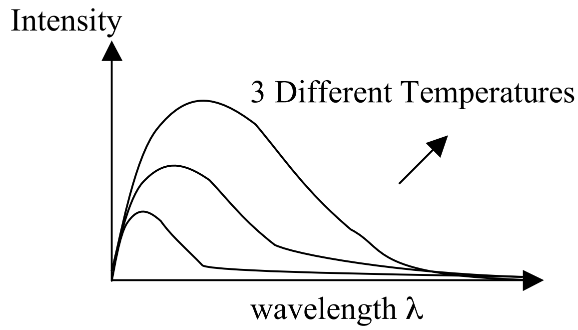


Image from:
http://map.gsfc.nasa.gov/m_uni/uni_101bb2.html

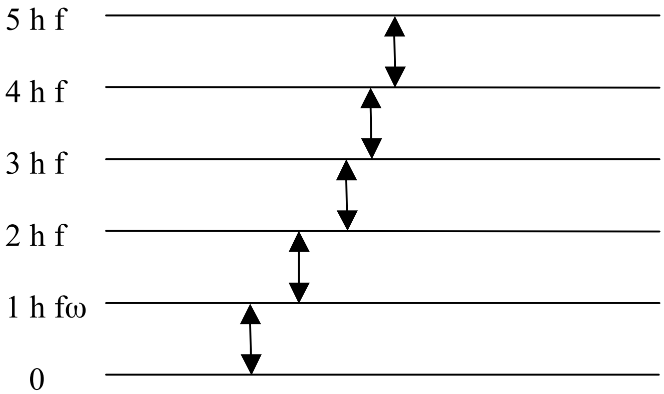
Blackbody Review

As T ↑   λ ↓
As T ↑   total Intensity ????↑
As T ↑   ν ↑



"Quanta"   L9p1

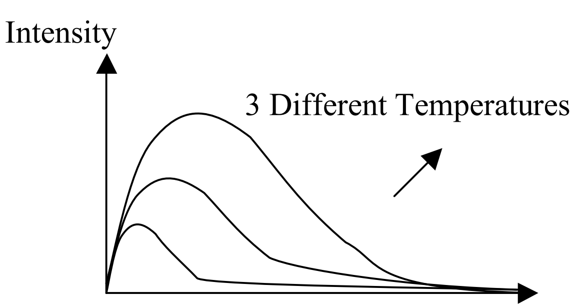
- M. Planck, 1900
- Assume radiation within the cavity is from "atomic oscillators"
- Then make 2 assumptions
    - The Energy of each oscillator can only have certain discrete values
      E = n h f   n = quantum number; h = Planck's constant, f = frequency
        - So each oscillator can have quantum states {0, 1hω, 2hω, 3hω, …}
    - Oscillators emit or absorb energy only when making transitions from one quantum state to another
      $\Delta E = m\,h\,f$ ($m = n_2 - n_1$)

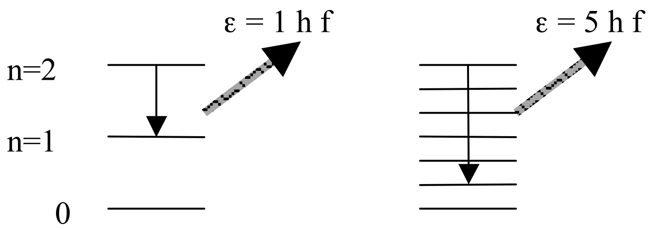


Energy level diagram

- If we now calculate the energy emitted by a black-body
    - Don't equally distributed Energy accross various λ bins (equipartition)
    - We must weight each λ bin according to a well-defined distribution that governs the occupation of higher energy states (This is the Boltzman distribution law) The weighting factor is ($e^{-\varepsilon/kT}$).

L9p2





Longest wave length shorter wave length

<u>Short wave length</u>

- Large energy separation
- Low probability of excited states
- Few downward transitions

<u>Long wave length</u>

- Small energy separation
- High probability of excited states
- Many downward transitions

Planck model L9p3

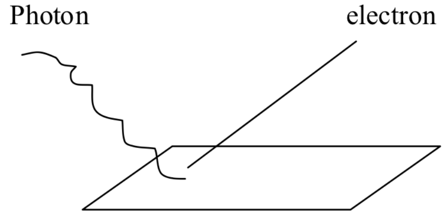
- Average energy is associated with a given λ
  - Product of E of transitions & factor related to probability of transition occurring
  - As energy levels further apart at shorter wavelength probability of excitation decreases
- At low frequencies, close together energy levels
  - $e^{-\varepsilon/kT}$ is large
  - many contributions, but each at low energy
- high frequency
  - energy levels far apart
  - Since ΔE is large, $e^{-\varepsilon/kT}$ is small, low probabiltiy of excitation
- Theoretical expression agrees with observation (the shape agrees)

  $I(\lambda,T) = 2\pi hc^2\lambda^{-5}(e^{hc/\lambda kT}-1)^{-1}$
  - "h" is then adjusted to get a good fit
    "h" is fundamental constant of nature
    h=6.626×10-34 J·s
  - Most people at the time (including Planck) thought this just a mechanical constant
  - Einstein tied this further to light
    - Quantization is a fundamental Property of light
      - <u>Photon</u>

Photoelectric Effect L9p4

- Extend quantum concept to E & M waves
- Assume light is a stream of quanta: photons

- Each photon has energy $E = hf$



- Each photon, when incident on plate,
    - Gives all energy to a single electron in metal
    - ∴ not continuous process, but discrete
        - You either liberate a photoelectron or you don't.
        - These electrons have maximum kinetic energy $KE_{max}=hf-\Phi$
            - Phi is work function needed to get electron unbound
              Varies by material
    - ∴ No dependence of KE on light intensity
    - ∴ No time delay between time of light incident and electron ejection
    - ∴ Dependence of electron ejection on light frequency since $E_\gamma>\Phi$.
    - ∴ Dependence of KE on light frequency

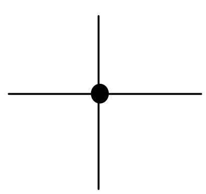
Photons and EM waves L10p1

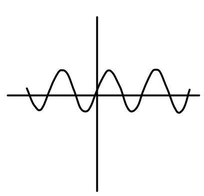
- Photons have frequency. f, wave-like
    - How can electrons be waves & particles?
        - Have Energy & momentum for a particle
        - Interfere and diffract for a wave
    - Cannot use these "classical" pictures to describe light adequately
        - We use both complementarily (because it is all we have).
            - Principle of Complementarity

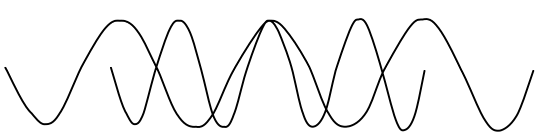
Wave properties of particles

- De Broglie postulate
    - Perhaps particles have, in analogy with light, wavelike properties
    - for example: $\lambda = h/p$ (light)
        - could also be for particles
            - $\lambda = h/mv$ (more generally, including relativistic particles: $\lambda = h/\gamma mv$)
                - de Broglie wavelength
            - $f = E/h$ (E = total energy of the particle)
    - relating particle & wave properties
    - accidentally confirmed by observation of diffraction: Davisson-Germer experiment
    - used for electron microscope
        - high Energy electrons, better resolution
        - $\lambda_e^s < 0.01\ \lambda_{\gamma vis}$

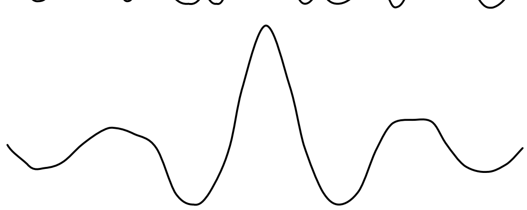
The Quantum Particle L10p2





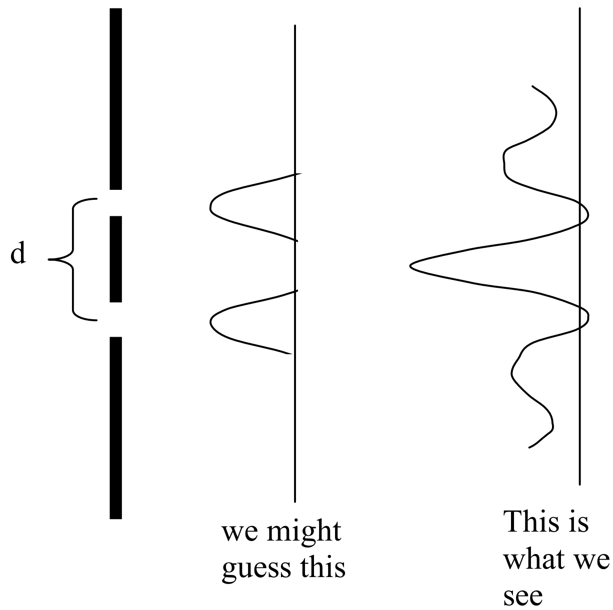


- Ideal particle: localized in space, zero size
- Ideal wave: single frequency, infinitely long
    - Unlocalized in space
- Take two waves of same phase & different frequency.
    - A somewhat preferred locale (or set of locales at best)
- Can construct superposition such that destructive interference everywhere but at $x=0$
  so have a particle with a location.



Wave packet

Double Slit Experiment L10p3



$d \sin \theta = \pi/2$ (minimum)

Consider a barrier with two slits in it in front of a screen. What happens when we send electrons at this barrier & screen? Some of the electrons will go through the barrier's slits & hit the screen & be recorded.
each electron goes through and registers at one location on screen.
If we send many electrons at the screen we'll begin to see them pile up in some places more than others (You might guess the pile up will be directly in front of the slits). but the probability of arriving at a given spot is determined by interference predictions! (this is a surprise, means each electron actually goes through BOTH SLITS!)

- if one slit covered
    - just one peak in front of the uncovered slit
    - we loose the interference pattern
- interpretation: an electron interacts simultaneously with both slits
    - if we try to determine which slit an electron goes through (by covering a slit) the interference pattern is destroyed. (Just like with Photons)

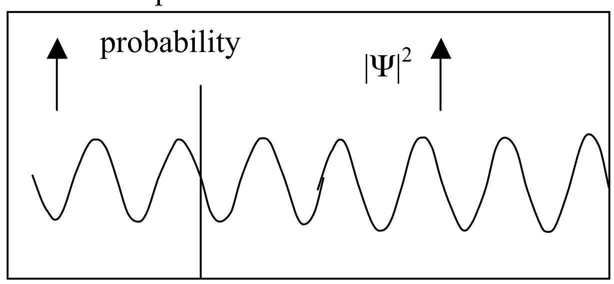
The Uncertainty Principle L10p4

- We usually have experimental measurement uncertainties
    - Classically these are not a fundamental problem & could theoretically be reduced to zero
    - Quantum mechanics does not allow this.
-
- If a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of it's x- direction momentum is made with uncertainty Δp_x, the product of these uncertainties will never be less than $\hbar/2$
  $\Delta x \, \Delta p_x \geq \hbar/2$.
    - similarly: $\Delta E \, \Delta t \geq \hbar/2$
    - Cannot measure momentum & position infinitely well, or well simultaneously
        - Not experimental uncertainty: from quantum state of matter.
- To know momentum precisely, by de Broglie $\lambda = h/p$, means we know λ precisely, as a pure λ, not superposition of many λ needed to localize the particle.
- To know x precisely, requires a large number of λ's, so p cannot be known.
    - Equations indicate energy conservation can be violated for time Δt.

Quantum Mechanics L11p1

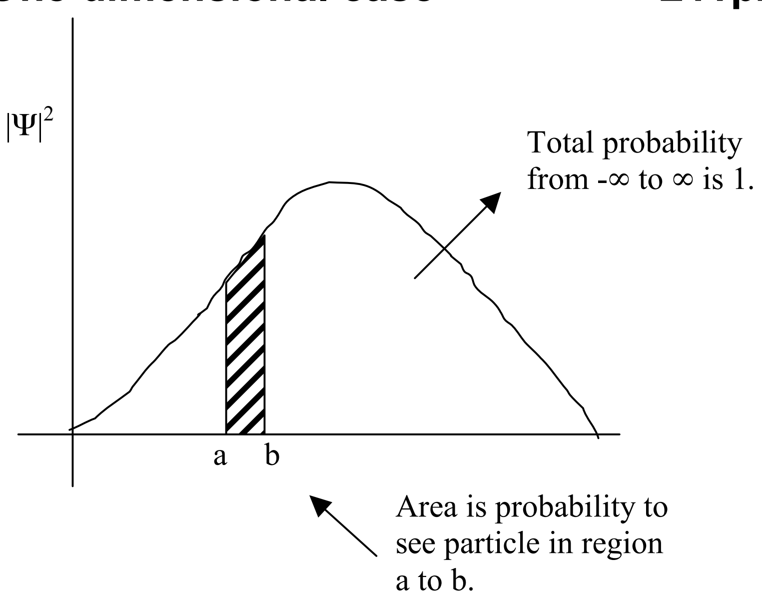
Probabilistic Interpretation in QM

- Matter and EM radiation waves & particles
  - "implies" probabilistic nature to being a particle
    - Particles in places more often when intensity of wave highest
    - For de Broglie particles
      - have "probability function" or "wave amplitude" Ψ.
      - contains all information we can know about a particle
  - Wave Eq. (Schrödinger Equation) 1926
    - Wave function changes in space & time
    - Probabilistic interpretation



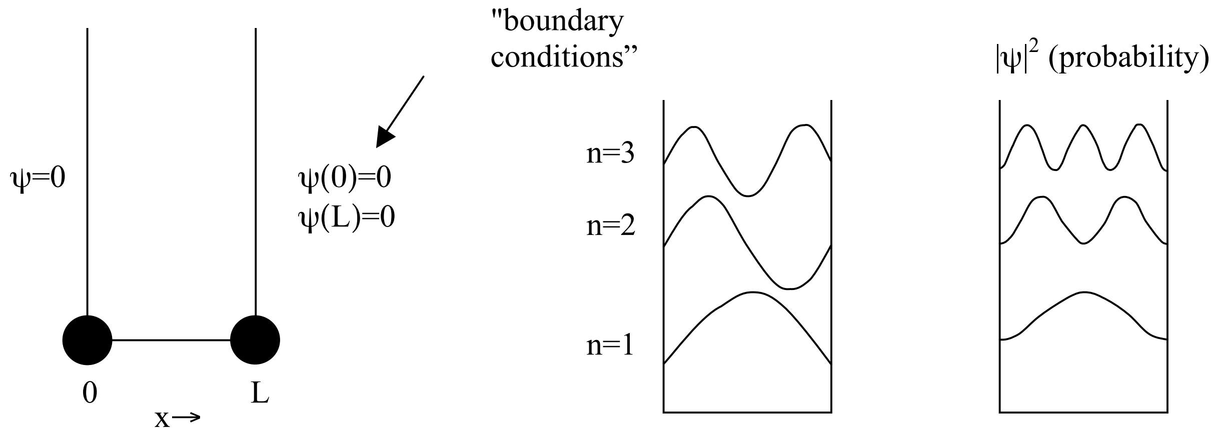
  - When have particle with very well defined (precise) momentum, it's spread through space.
    - Probability density, ψ^2.
      - Probability to find particle in volume element around some point.

One dimensional case L11p2



Particle in a Box

- Particle moving back & forth in classical physics with impenetratble walls
  - Can have any momentum or energy
- In quantum physis
  - zero probability at walls.

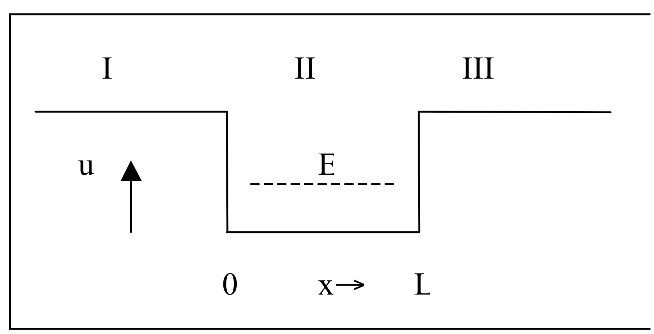


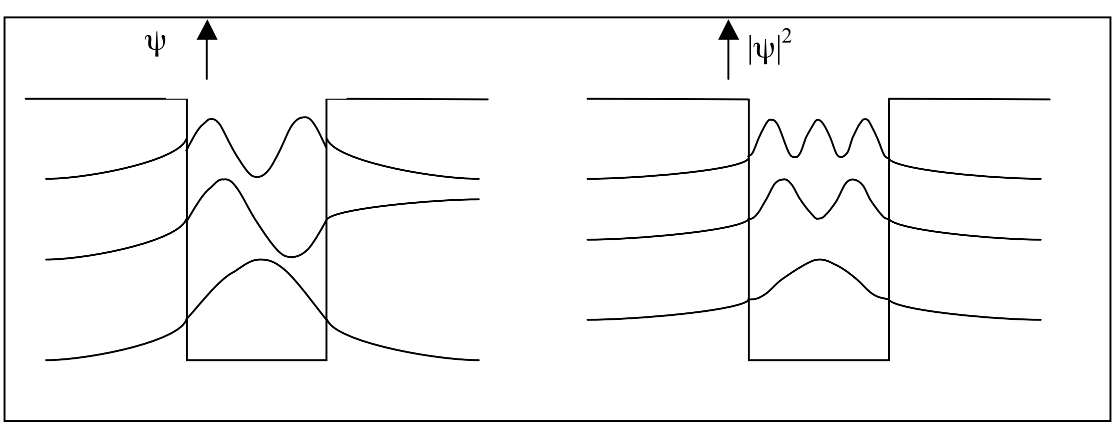
Since only certain wavelengths 'fit', the energy levels of the trapped particles are discrete.

Therefore, **energy is quantized!**

Boundary Conditions & Finite Potential Well L11p3

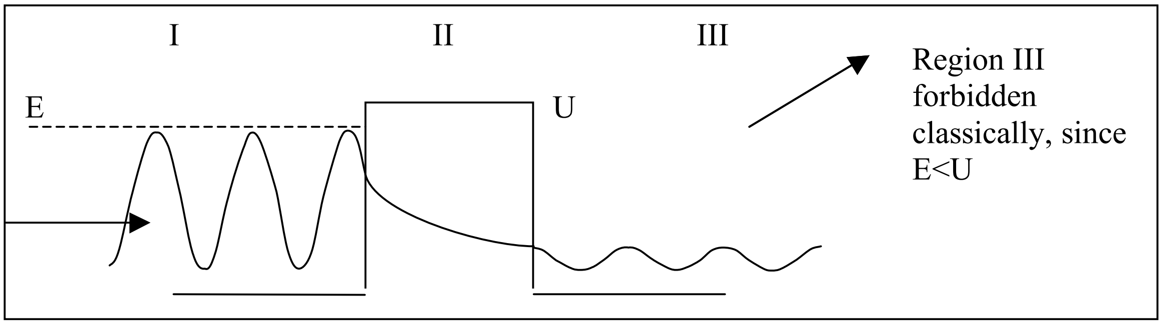
- Interaction of a particle with its environment is represented by potentials and
  - One or more "boundary conditions"
  - If a particle is restricted to a finite region of space
    - Quantization of E of system results
  - Soulstions to wave equation must be continuous → must join smoothly at boundaries
- If walls not quite impenetrable
  - Ie. $U < \infty$
  - Classically, if $E < U$ the particle will always be in the well
  - QM
    - Probability density ≠0 in regions I & III
    - Because energy of system uncertain can occasionally have $E > U$
    - In II, sinusoidal → but boundary conditions requiree continuity





Tunneling L11p4

-
- Consider moving particles with energy E
  - Heading toward a Potential barrier of height U>E
  - Finite width.



  - Sinusoidal in Region I
  - Exponential in Region II
    - But there is a non-zero probability of finding the particle to the right side of the barrier.
  - Sinusoidal in Region III with reduced amplitude (also non-zero)

∴ The particle can be anywhere!

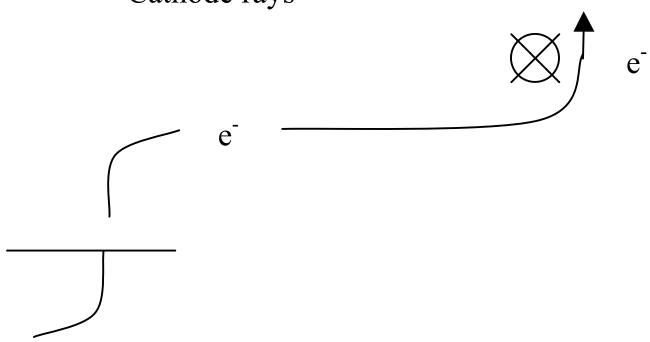
Thus we get tunneling from I□III, which is completely contrary to classical physics.

Discuss JJ Thompson & The Electron   L12p1

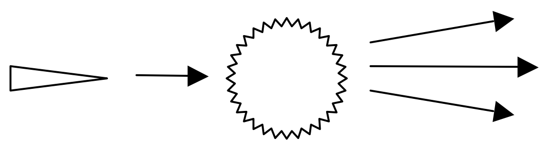
Hydrogen Blamer, Lyman, Paschen series of lines

Atomic Structure

- Cathode rays



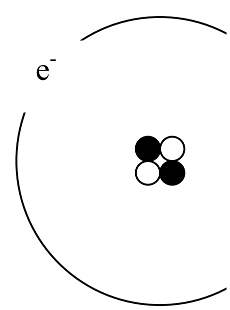
  - Measure properties of electron (e/m)
    - Assume atom is like fragments of onion (electrons) in a meatball!
      - Onion weighs nothing, meatball has no heavy components (uniform mass distribution in meatball).



Expected result

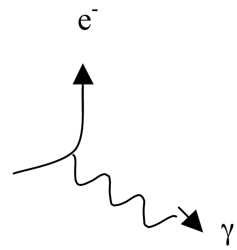
  - Scattering experiment (Rutherford)

    α □ [ ] thin gold foil   (ie, the non onion part of meatball is massive)
    - Expect same thing
    - Except sometimes, α comes back at you!
      (bullet reflected back by meatball)
      ("cannon ball stopped by a tissue")
    - So most mass of atom must be concentrated at center of an atom
      "Planetary model of atom"



Bohr Model   L12p2

- Problem
  - When accelerated e^-'s, emit energy
    - Accelerate means change velocity



- So e^-'s spiral into atomic nucleus
  - atomic structure not stable
- Ideas
  - Circumference of e^- orbit = n λ_e.
    - In these cases, no emited radiation
    - Only certain λ's permitted to create an integer multiple
    - Ie. e^-'s have discrete energy levels
  - Radiation can be emitted when e^- makes transition from one level to another ($\Delta\lambda$)

"transition"

- $E_i - E_f = h f$ where f is frequency of γ emitted.
- Give roughly correct description of spectral lines for Hydrogen
  - Heavier elements & fine structure not accounted for
  - Electrons in well defined orbits not relained