Diffraction Patterns

- Diffraction refers to fact that wave spreads beyond shadow of aperture
- We don't just see wide illuminated region.
- Secondary maxima

- Illuminate penny
- Get bright spot

- Assume rays reaching screen approx. parallel

- Fraunhofer diffraction pattern

- A slit has a non-zero width
- Each point in slit is a source of light waves
- As if multiple slits, all interfering

- Consider a slit with two halves
- A path difference between 1/4, 1/2

- If \( \frac{\lambda}{2} \sin \theta = \frac{1}{2} \): destructive interference waves from upper half of slit destructively with lower half
- First min.

- If divide into 4ths, 6ths...\[
\sin \theta = m \frac{\lambda}{a} \quad (m = 2, 3, 4, ...)\]
- Where have dark regions
Intensity - Single Slit

Each subsection of slit, $dy$, is source of coherent radiation.

For a point $P$, gives some $\Delta \delta$.

Adjacent $\Delta y$ is phase difference by $\Delta y \sin \theta$.

$\Delta \delta = \frac{2\pi}{\lambda} \Delta y \sin \theta$

Total phase difference across slit $\beta = N \Delta \delta = \frac{2\pi N \Delta y \sin \theta}{\lambda}$.

When $\Delta y = 0$.

$\beta = 0 \rightarrow \beta = \frac{2\pi N \Delta y \sin \theta}{\lambda}$.

For $1$st max, $\beta = 0 \rightarrow \frac{2\pi N \Delta y \sin \theta}{\lambda} = \frac{2\pi}{\lambda} \sin \theta = \frac{\lambda}{2}$.

For $1$st min, $\beta = \frac{\lambda}{2} \rightarrow \frac{2\pi N \Delta y \sin \theta}{\lambda} = \frac{\lambda}{2} \sin \theta = \frac{\lambda}{2}$.

For $2$nd max, $\beta = \frac{2\lambda}{2} \rightarrow \frac{2\pi N \Delta y \sin \theta}{\lambda} = \frac{3\lambda}{2} \sin \theta = \frac{\lambda}{2}$.

$\sin \theta_{\text{dark}} = \frac{\lambda}{2\Delta y}$.

Consider limit of $\Delta y \to 0$ (dy)

- All $\Delta \delta$'s delineate arc in phase diagram.

$\sin \frac{\beta}{2} = \frac{E_0/2}{R}$

- Geometrically, $E_0$ is arc length $4R$.

$\frac{E_0}{R} = \sin \frac{\beta}{2} \Rightarrow \frac{E_0}{R} = 2 \left( \frac{E_0}{R} \right) \sin \left( \frac{\beta}{2} \right) = E_0 \sin \left( \frac{\beta}{2} \right)$.

$I = I_{\text{max}} \left[ \sin \left( \frac{\beta}{2} \right) \right]^2$.

When $\beta = \frac{\lambda}{2} = \pi n$, $\sin \beta = \sin \frac{\lambda}{2}$.

$\beta = \pi n \rightarrow$ minima.

$I = 0, 1, 3, 3...$
### Two slit diffraction

- With two slits, diffraction from each slit interference between 2 slits

$\frac{I_1}{I_0} \propto \frac{\sin \left( \frac{m \Delta \phi}{2} \right)}{m} \frac{\sin \left( \frac{m \phi}{2} \right)}{m}$

$I_0 \cdot \text{single slit} \times I_2 \cdot \text{2 slit}$

peak $I_2$ is roughly constant

### Resolution

- 2 sources of light
- ea. produce separate image/splot + diffraction pattern
- angular separation $\theta$

Rayleigh's criterion:
- 2nd central image of $\theta$ falls on first max. of $\theta$
- termed "just resolved"

$\sin \theta = \frac{1}{2} \lambda \Rightarrow \text{min}$

$d_{\text{min}} = \frac{1.22 \lambda}{D}$

$D$ = diam. of aperture