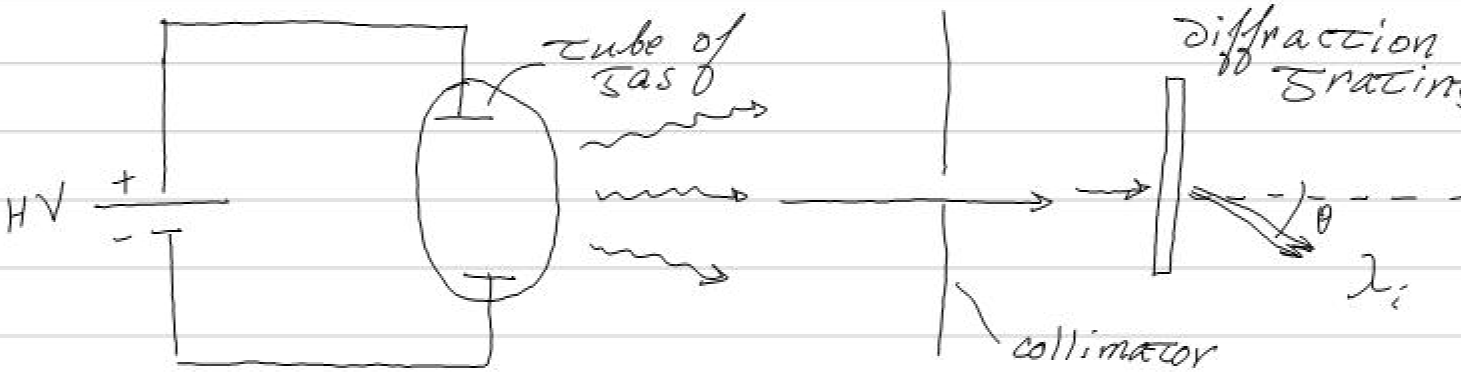
Line Spectra

Placing gas in a high electric field
- a sudden discharge of electrons (breakdown like lightning)
- as electrons collide with atoms in the gas → excite them
- excited atoms will deexcite by emitting EM waves



What are the properties of the emitted light?

- seperate λ_s via diffraction grating

$$\boxed{d \sin\theta = n\lambda}$$

longer λ, larger angle deflection

- white light → grating produces a full spectrum (rainbow) eg.
  - → all colors, purple → red
- → in case above
  - only certain θ^s (λ^s) have light
  - different set of "emission λ^s" or "lines" associated with each element
- unknown set of lines → Helium Discovery

Pattern of Spectral Lines

Correlation of set of emission lines with each chemical element suggests something about atomic structure

Take Hydrogen (turns out to be simplest case)

Empirical law describing pattern

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{k^2} \right) \qquad n = 1, 2, 3, \ldots \quad k > n$$

Rydberg constant $= 1.0968 \times 10^7 \, m^{-1}$

early 1900s

- $n=1$ Lyman series (UV)
- $n=2$ Balmer series (visible λs)
- $n=3$ Paschen " " (infrared)

Where does this come from?

Classical Physics has no way to understand.

Key clue to new Physics of Quantum Mechanics

→ quantization of energies

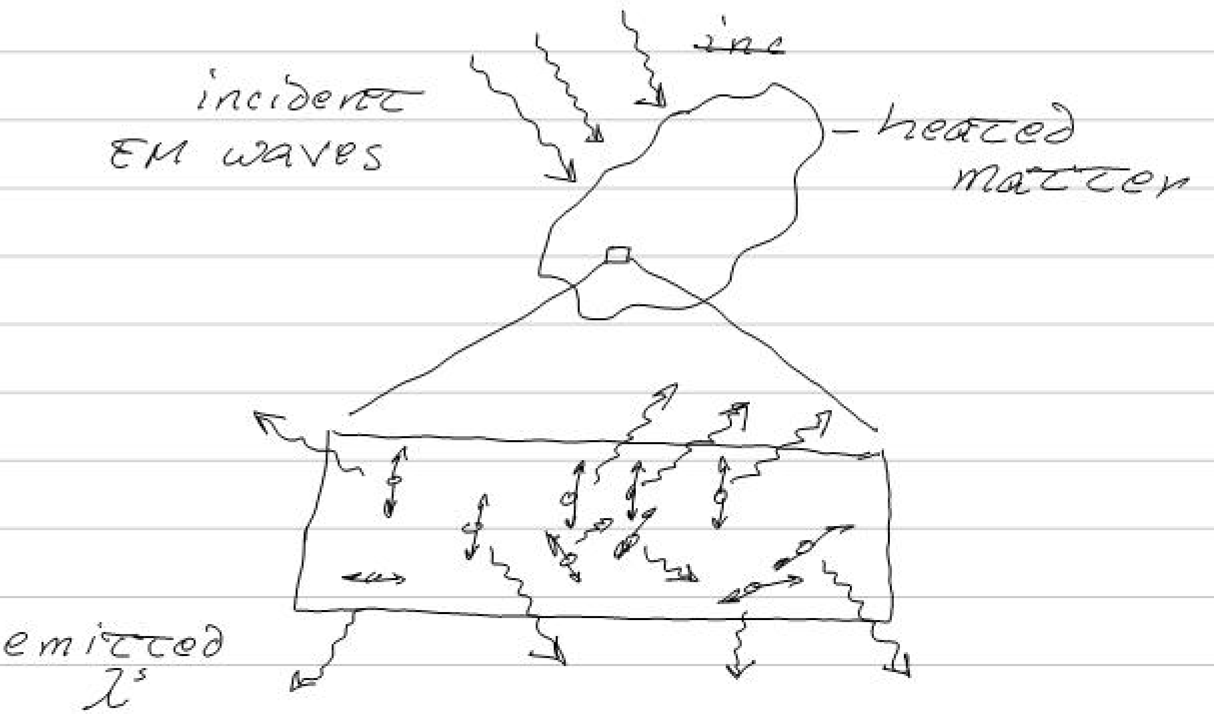
Thermal Radiation:

Metallurgy: metal implements for millenia (swords, wheels, etc.)

→ when matter heated, it radiates with a characteristic λ

$T = 600°C \rightarrow$ red λ

→ get "bluer" as Temperature rises



This is neither reflection nor physics of spectral lines

→ all materials do it in a broad continuous distribution of λ^s

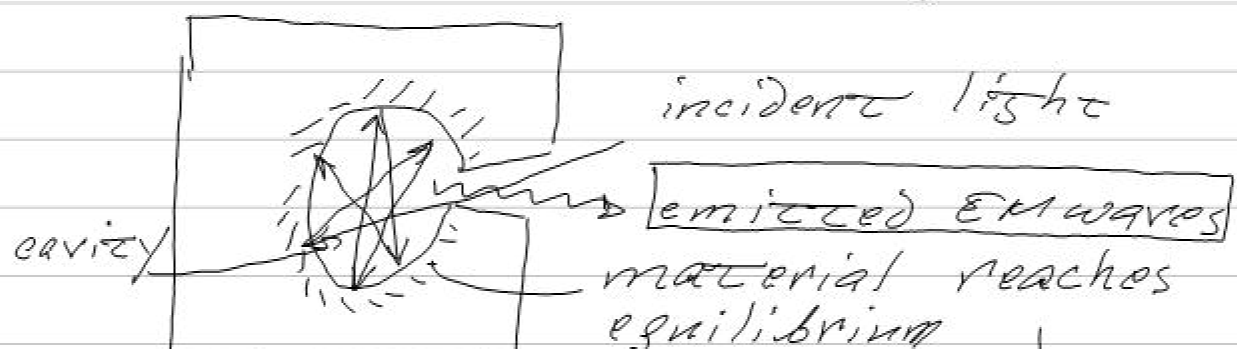
The Blackbody

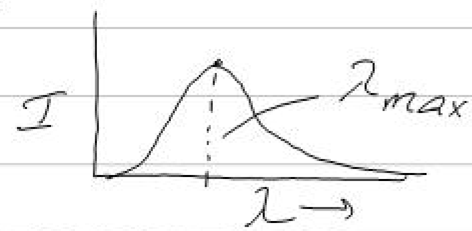
Idealize the situation; consider an object which absorbs 100% of incident radiation (perfect absorber)

- create scenario where heated by incident EM waves
  - → wait till its temperature stops changing

Thermal Equilibrium

rate of absorption = rate of emission





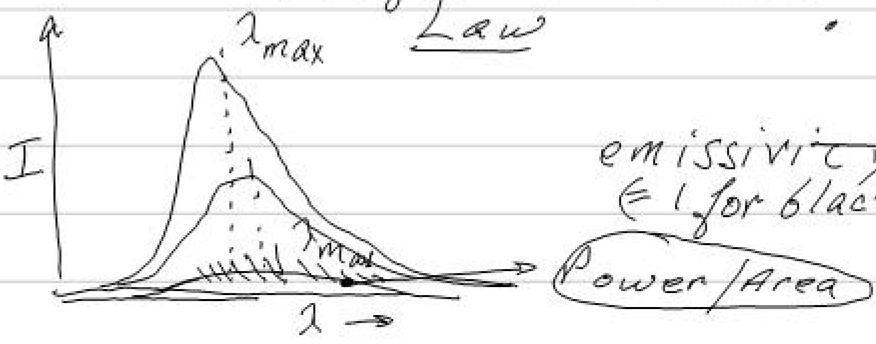
Observations

Wien's Law: $\lambda \uparrow T \downarrow$ $\quad \boxed{\lambda_{max} T = 2.9 \times 10^{-3}\, m \cdot K}$

Stefan-Boltzmann Law: $\frac{Power}{Area} = \int_0^\infty I(\lambda, T)\, d\lambda = \boxed{e\, \sigma T^4}$

e: emissivity ($= 1$ for blackbody)

σ: $5.7 \times 10^{-8}\, W/m^2 \cdot K^4$

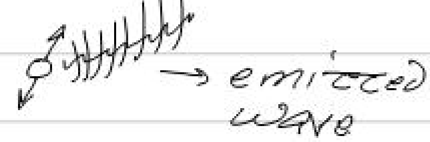


Classical Models

- Radiation absorbed somehow by atoms in material

absorbed wave

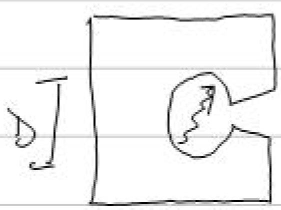
- atoms, known to contain electric charges, start to oscillate/vibrate



$\therefore$ acceleration of charge $\Rightarrow$ EM waves

→ consider our apparatus

- EM waves are standing waves in cavity $\lambda \leq \delta$



→ huge # of short λ's 'fit' in cavity

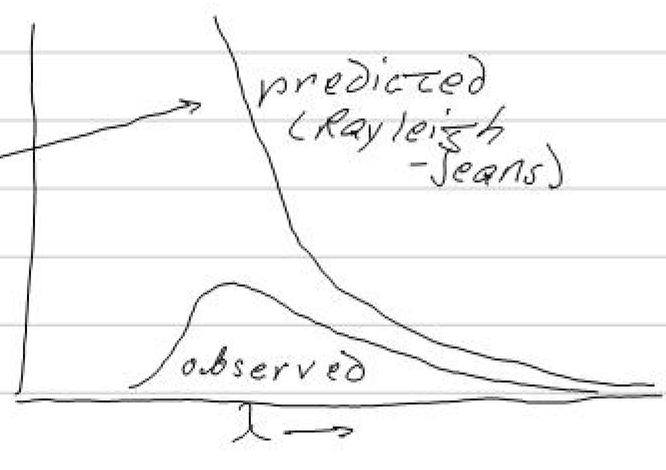
$\therefore \infty$ # oscillatory modes as $\lambda \to 0$

Equipartition theorem of thermodynamics

'equal energy kT to each possible mode'

Result:

"ultraviolet catastrophe"



The Quantum

Solve the problem by limiting the number of Energy levels:

- only certain λ's + Energies satisfying $E = n h \nu$

$n = 1, 2, 3, ...$ — "Planck" constant — frequency

Planck's constant $= h =$
$$= 6.6 \times 10^{-34}\ \text{J}\cdot\text{s}$$

Because Energy levels have $E = nh\nu$,
- oscillators absorb + emit in discrete multiples of

$$\boxed{\Delta E = h\nu}$$

Under this assumption, one gets

$$\boxed{I(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}}$$

- the form of this function agrees well with experimental results (h is free parameter)

Planck did not fundamentally accept the idea of quantization and tried to fix it for a long time.

Problem

Estimate the power radiated by a) a basketball @ 20°c, and b) the human body @ 37°c.

a) Power/area $= \sigma T^4$

$= (5.6705 \times 10^{-8} W/(m^2 \cdot K^4))(293\ K)^4$

$= 420\ W/m^2$

Power $= (420 W/m^2)(4\pi r^2)$ $\quad r = 11 cm$

$= \underline{69 W}$

b) Person is ~ 1.75m tall cylinder

radius $= 0.15 m$

$\therefore$ area $= 2\pi r h = 1.65 m^2$

Power $= (\sigma T^4) \times (Area)$

$= (5.67 \times 10^{-8} W/m^2 \cdot K^4)(310 K)^4 (1.65 m^2)$

$= \boxed{864.4 W}$

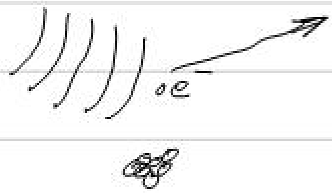
Photoelectric Effect:

It was slowly learned that e^-s bound in atoms

→ weak binding in metals, hence conduction

3 ways e^- can be extracted from surface of metal

- heat (thermionic emission)
- collisions (secondary emission)
- E field: "pulls" e^- out of atom
- photoelectric: incident light



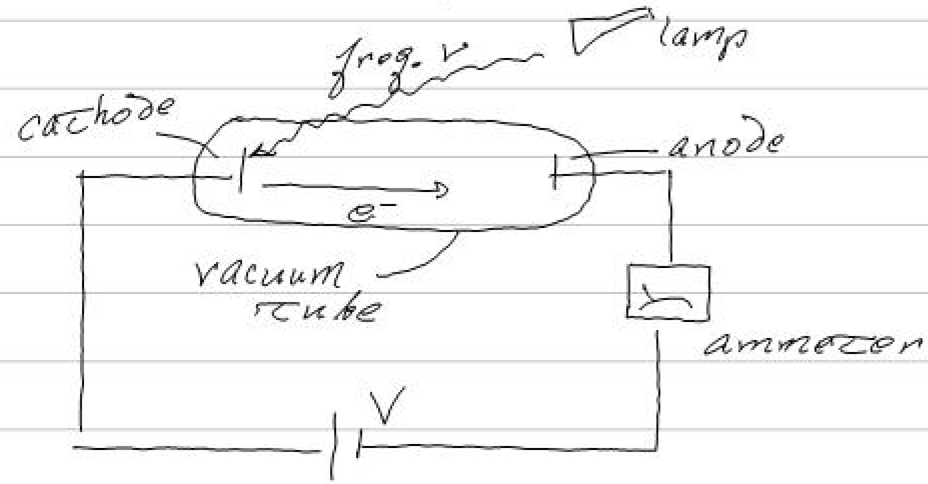
If enough incident E

e^- can reach sort of "escape velocity"

$K.E._{minimum}$ needed for escape

ϕ = work function = min. Binding E

Experimental Setup:



A lamp w/light of freq. ν is shined on cathode.

In classical case:

- total energy in a light wave increases as light intensity increases

$$\cancel{I}_{light} = S_{avg} = \frac{\mathcal{E}_{max}^2}{2\mu_0 c}$$

$$= c\, u_{avg}$$

average E density

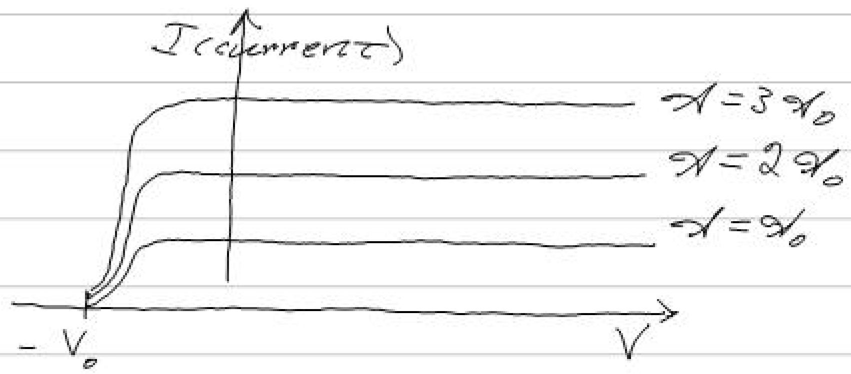
So energy imparted by EM-wave

$\underline{\underline{\propto \text{ Intensity}}}$ ($\propto$)

- no dependence on ν of light

Dependence of Current on Voltage

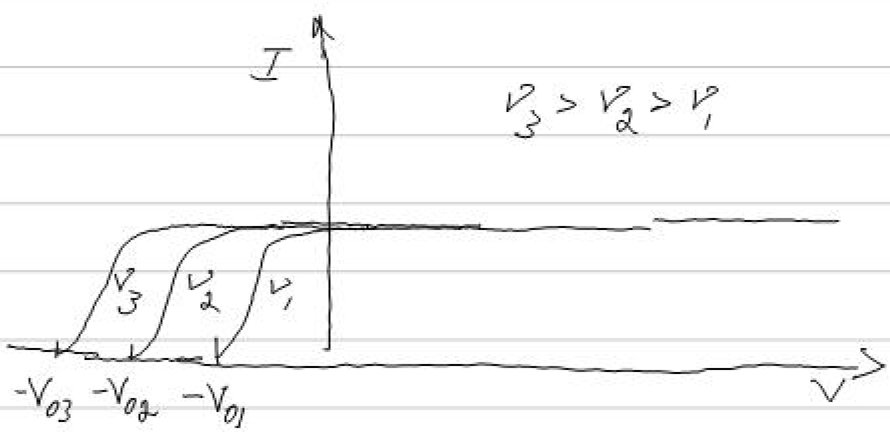
(A) Hold frequency, ν, constant & change intensity, α:



$\underline{\underline{V_0}}$ is a negative potential which can stop photo current (i.e. work $= -$ K.E.)
→ observed to be constant vs. α

∴ $\boxed{\text{K.E. of } e^{-}\text{s independent of } \alpha}$!
(problem for Classical E&M)

(B) Hold α constant, & change ν:



If a larger ν, the stopping voltage increases

$\boxed{\text{∴ max. K.E. of } e^{-}\text{s increases}}$!
(problem for Classical E&M)

Other Distributions

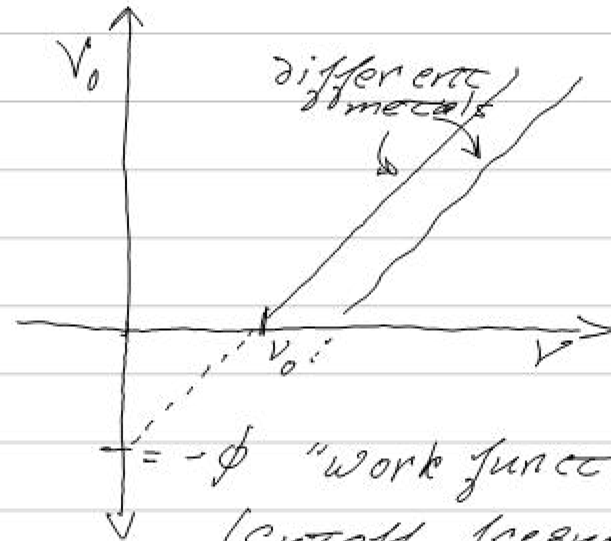
Ⓒ Threshold or Cutoff frequency:

Plot stopping voltage vs. frequency:

- below some ν_0, no e^- emitted

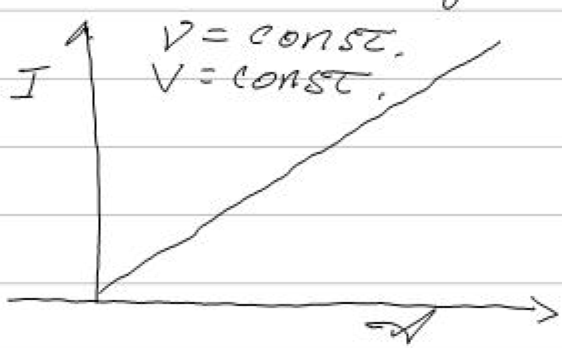
  ν_0 does not depend on $\mathcal{I}$

  → depends on material



(cutoff frequency has no analog in classical E+M.)

Ⓓ Production of Photocurrent



# of photoelectrons (i.e. "photocurrent") increases with $\mathcal{I}$

(this is consistent with classical E+M)

Ⓔ Timing:

- As soon as light is incident (i.e. 3×10^{-9} s), full current is observed
  - → even @ very low light intensity levels

  (Classically, @ low light Δt should become long.)

Role of the Quantum, Part Deux:

Observations are not consistent with classical picture.

Hypothesis (Jr. Einstein): light itself (i.e. the EM field) exists in discrete units, termed "photons" ('γ')

- like localized burst or concentration of EM wave

$$\boxed{E_\gamma = h\nu}$$

ν: frequency of wave

h: Planck's const.

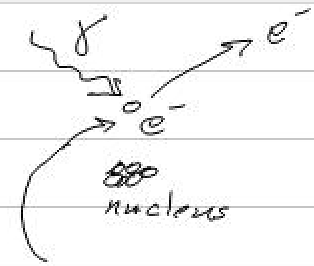
This generalizes Planck's idea for blackbody radiation

Photons must be emitted or absorbed in toto, not partially

- in vacuum, velocity is $\boxed{c = \lambda\nu}$

So while it's a wave, it's got particle-like properties

Photoelectric Effect Reconsidered



A γ (photon) gives all energy to e^-

- e^- can loose some E as passes thru material

We only care about maximum K.E. of e^-

$$E_\gamma = h\nu = \phi + K.E._{max} = \phi + \frac{1}{2} m v_{max}^2$$

before ⋮ after collision

$$\boxed{h\nu - \phi = \frac{1}{2} m v_{max}^2 = e V_0}$$

(A) K.E. of e^- not dependent on α! α has no meaning @ single γ level, either have a collision with e^-, or don't

(B) K.E of e^- depends on ν: yes, $V_0 = \frac{1}{e}(h\nu - \phi)$

(C) Linear relation of V_0 + ν: $V_0 = \frac{1}{e}(h\nu - \phi)$
→ slope gives measure of 'h': It took 10 yrs, but confirms value same as from blackbody radiation.

(D) I vs. α: more γ's can eject more e^-'s
∴ more photo current

(E) $\Delta t \sim 0$: there is no "warm up" period required to free e^-: just travel time of γ's from cathode to anode.

Problem:

$\lambda = 580\,nm$

The minimum light to register with human eye is $\sim 100\,\gamma^s$.

How much energy is present?

$$E_\gamma = h\nu = hc/\lambda$$

$$= (6.63 \times 10^{-34}\,J\cdot s)(3 \times 10^{8}\,m/s)/(580\,nm)$$

$$= 3.43 \times 10^{-2} \times 10^{-34} \times 10^{8}/10^{-9}\ J$$

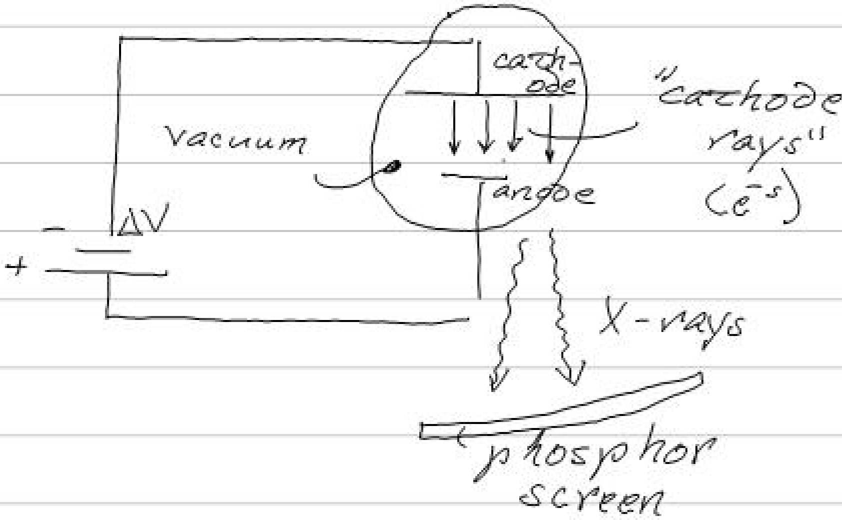
$$= \underline{3.43 \times 10^{-19}\,J} \text{ per photon}$$

Since energy is quantized, for a monochromatic source

$$E_{total} = 100\,E_\gamma$$

$$\boxed{= 3.43 \times 10^{-17}\ J}$$

X-ray Production



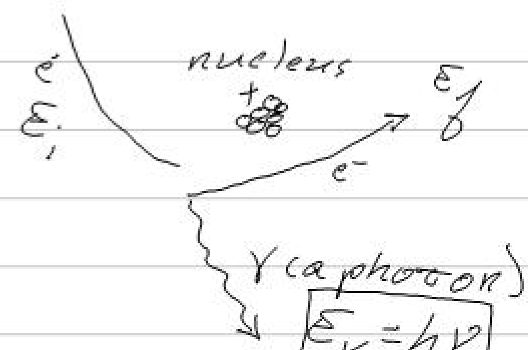
An electric field draws "cathode rays" (e^-s) so that some are incident on the glass tube.

"X-rays" produced

How do we know about X-rays?
→ show up on a phosphor screen
→ their paths unaffected by B-field

What's going on?

When e^- passes near atomic nucleus (which has intense E field)



Bremsstrahlung

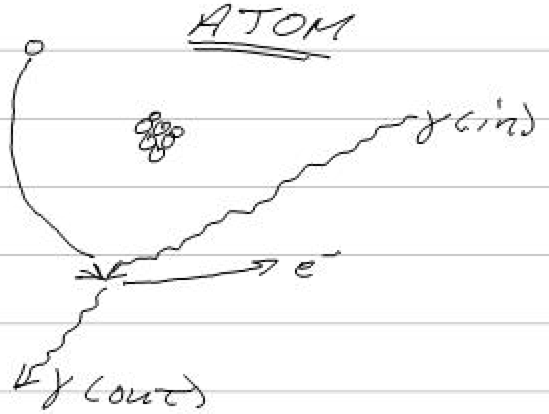
$$\boxed{\mathcal{E}_f = \mathcal{E}_i - h\nu}$$

fr. conservation of energy

X-rays used for many things

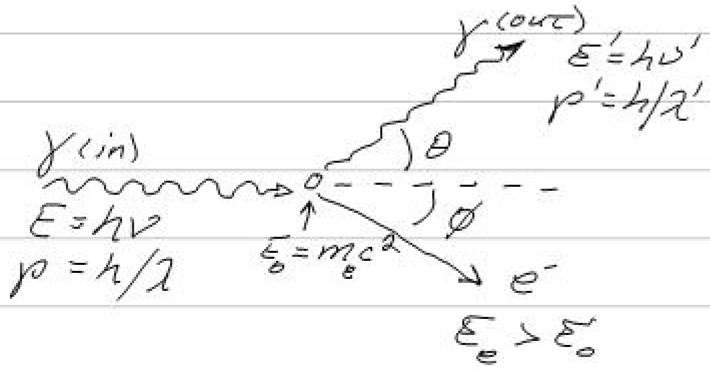
Medical imaging in Crimean War (5 mo.s after discovered)

Compton Effect



$\Longrightarrow$

Go To e^- rest frame:



When a γ enters vicinity of an atom

<u>Classically</u>: EM wave causes same λ oscillations in e^-s
→ e^-s then radiate @ same λ
(Thomson scattering)

<u>Observation</u>:
- some 're-radiated' waves observed to have longer λs than initial wave
- largest $\Delta\lambda$ when $\theta = 90°$

discovered by Gray

<u>Compton Effect</u>

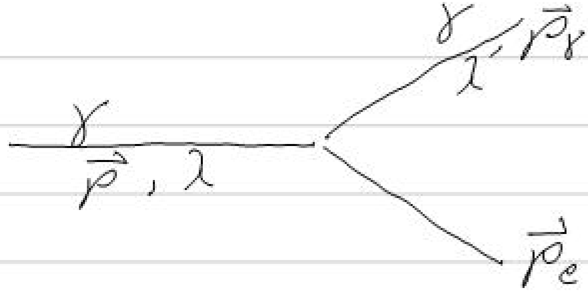
???

Compton's Hypothesis:

Have a classic 2-body interaction

$1\gamma \rightarrow \leftarrow 1e$

Not aware interactions with all e^{-}'s in material



$\vec{p} = \vec{p}_\gamma + \vec{p}_e$

Apply conservation of Energy & Momentum.

$$p_x: \ \frac{h}{\lambda} = \frac{h}{\lambda'} \cos\theta + p_e \cos\phi$$

$$\frac{h^2}{\lambda^2} = \frac{h^2}{\lambda'^2} \cos^2\theta + 2\frac{hp_e}{\lambda^2} \cos\theta \cos\phi + p_e^2 \cos^2\phi$$

$$p_y \Rightarrow \frac{h}{\lambda'} \sin\theta = p_e \sin\phi$$

$$\frac{h^2}{\lambda'^2} \sin^2\theta = p_e^2 \sin^2\phi$$

Adding these gives

$$\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2}\sin^2\theta = \frac{h^2}{\lambda'^2}\cos^2\theta + \underbrace{p_e^2\cos^2\phi + p_e^2\sin^2\phi}_{= p_e^2}$$
$$+ 2\frac{h p_e}{\lambda'}\cos\theta\cos\phi$$

Using $p_e\cos\phi = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta$ and bringing p_e to the left

$$p_e^2 = \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2}(1-\cos^2\theta) - \frac{h^2}{\lambda'^2}\cos^2\theta - \frac{2h}{\lambda'}\cos\theta \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)$$

$$= \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} \cancel{- 2\frac{h^2}{\lambda'^2}\cos^2\theta} - \frac{2h^2}{\lambda\lambda'}\cos\theta + \cancel{\frac{2h^2}{\lambda'^2}\cos^2\theta}$$

$$= \frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta$$

Using conservation of energy

$$E_e^2 = [\underbrace{\cancel{h\nu}}_{h/\lambda} + m_e c^2 - \underbrace{\cancel{h\nu'}}_{h/\lambda'}]^2 = m_e^2 c^4 + p_e^2 c^2$$

$$\left[hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + m_e c^2\right]^2 = m_e^2 c^4 + \left[\frac{h^2}{\lambda^2} + \frac{h^2}{\lambda'^2} - \frac{2h^2}{\lambda\lambda'}\cos\theta\right]c^2$$

$$h^2 c^2\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^2 + 2m_e c^2 hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + \cancel{m_e^2 c^4}$$
$$\left(\cancel{\frac{1}{\lambda^2}} - \frac{2}{\lambda'\lambda} + \cancel{\frac{1}{\lambda'^2}}\right) \qquad = \cancel{m_e^2 c^4} + c^2\left[\cancel{\frac{h^2}{\lambda^2}} + \cancel{\frac{h^2}{\lambda'^2}} - \frac{2h^2}{\lambda\lambda'}\cos\theta\right]$$

$$-\frac{h}{\lambda'\lambda} + m_e c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = -\frac{h}{\lambda\lambda'}\cos\theta$$

$$m_e c\left(\frac{\lambda' - \lambda}{\cancel{\lambda'\lambda}}\right) = \frac{h}{\cancel{\lambda\lambda'}}(1 - \cos\theta)$$

$$m_e c \, \Delta\lambda = h(1-\cos\theta)$$

$$\boxed{\Delta\lambda = \frac{h}{mc}(1-\cos\theta)}$$ Compton Effect

The Compton wavelength is

$$\lambda_c = h/mc = \underline{2.4 \times 10^{-3}\,nm}$$

- if $\lambda \gg \lambda_c$ (i.e. visible light), then $\Delta\lambda/\lambda$ is very small
  - so need X-rays or γ-rays to observe effect

What's happening?

- when γ scatters off e^-, e^- considered unbound particle
  $m \to m_e$
- if e^- tightly bound, $m \to m_{nucl}$
  - γ essentially scattering off the whole atom

In other words, mass of nucleus is firmly attached to e^-

$\Delta\lambda$ much smaller

↳ Thomson scattering

Use of Special Relativity + Quantum
↳ convincing use of Modern Physics

Problem:

A 6keV γ scatters off a proton with angle 90°

$m = m_p$

$\theta = \pi/2 \rightarrow \cos\theta = 0$

$\Delta\lambda = h/mc \, (1-0)$

$= \dfrac{4.14\times10^{-15}\ eV\cdot s}{938.27\times10^{6}\ eV/c^2} \cdot c$

$= 4.41\times10^{-3}\times10^{-21}\ s/\cancel{c^{1}}\times c^2$

$= 1.47\times10^{-24}\times10^{-8}\ s^2 m^{-1}\cdot c^2$

$= \cancel{1.47\times10^{-23}\ nm}$!! $1.3\times10^{-15}\ m$

$= 1.3\times10^{-6}\ nm$

$6keV \Rightarrow = hc/\lambda$

$\underline{\underline{\lambda}} = \dfrac{4.41\times10^{-15}\ eV\cdot s \times 3\times10^{8}\ m/s}{6\times10^{3}\ eV}$

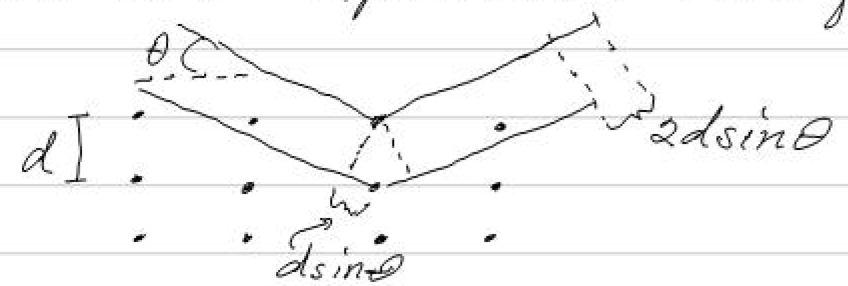
$= \underline{0.22\ nm}$

We have seen that EM waves can also behave like particles

wave properties

→ double slit experiment : interference

X-rays in crystals



The condition for constructive interference $\boxed{n\lambda = 2d\sin\theta}$ $(n=1,2,..)$

Bragg's law

Particle properties

- photoelectric effect
- Compton effect
- spectral lines (via orbital model)

$p = h/\lambda$ $E = h\nu$

Somehow light has both

waves : no localization..

particles : localization completely