CH 2 NEWTON'S LAWS

(2) $\Sigma \bar{F} = \frac{d\bar{p}}{dt} = m\bar{a}$ (IF m CONST ⇒ ~~ex~~: NOT LOSING FUEL)

(1) $\bar{v}$ = CONST IF $\Sigma \bar{F} = 0$

(3) OBJECTS EXERT EQUAL & OPP. FORCES

just say

COMMENTS

(1) SPECIAL CASE OF (2)

(1) & (2) ONLY 1/2 A LAW UNTIL $\bar{F}$ SPECIFIED

⇒ MAINLY SERVE AS DEFNS OF $\bar{F}$, m

ex (a) PICK REF. OBJECT W/ MASS m_0 ⇒ DEFINES UNIT OF MASS

(b) MEAS $\bar{a}$

(c) $\Sigma \bar{F} \equiv m_0 \bar{a}$ DEF'S $\Sigma \bar{F}$

(d) KEEP SAME $\Sigma \bar{F}$ BUT NEW m' ⇒ NEW $\bar{a}'$

$$\Rightarrow \quad m'\bar{a}' = m_0 \bar{a}$$

$$m' = m_0 (a/a') \quad \Rightarrow \text{DEFINES MASS}$$

There are other interpretations

"INERTIAL MASS" = RESISTANCE TO $\bar{a}$

⇒ AT FACE VALUE, SEEMS THIS IS ALL THAT'S THERE, BUT BECAUSE WE HAVE INTUITIVE NOTION OF $\bar{F}$ (ie CAN USUALLY TELL WHEN $\bar{F}$ IS PRESENT) THERE'S MORE:

(A) BREAK FROM ARISTOTLE:

$\bar{v}$ REQUIRES EXPLANATION ⇒ LOOK FOR CAUSE

BUT $\bar{v} = 0$ ⇒ NATURAL CONDITION ⇒ NOTHING TO EXPLAIN

(most of us are aristotelians before intro physics)

VS. NEWTON:

$\bar{a}=0$, $\bar{v}$ = CONST $\Rightarrow$ NOTHING TO EXPLAIN

$\bar{a} \neq 0$ $\Rightarrow$ LOOK FOR $\bar{F}$

CAN TEST THESE:

- IF I ELIMINATE ALL EXTERNAL INFLUENCES (ie $\Sigma\bar{F}$)
  WHICH IS CORRECT? REQUIRES SOME COMMON SENSE
- IF PURELY A DEFN, CAN'T TEST (BY DEFN)

(3) INERTIAL FRAMES & NEWTONIAN RELATIVITY

(1) PICKS OUT SPECIAL FRAMES OF REFERENCE

(a) FIND OBJECT w/ NO $\Sigma\bar{F}$ $\Rightarrow$ EXPECT $\bar{a}=0$

TRUE? I'M IN AN INERTIAL FRAME

(b) CONSIDER SOMEONE IN CAR SPEEDING UP
OR PLANE TURNING $\Rightarrow$
SAME OBJECT SEEMS TO HAVE $\bar{a} \neq 0$ BUT $\Sigma\bar{F}=0$

$\Rightarrow$ NOT AN INERTIAL FRAME

$\Rightarrow$ FICTITIOUS $\bar{F}$ DUE TO MY SYS OF REF ACCEL.

$\Rightarrow$ CAN TELL IF I'M ACCEL. IN NEWT. MECH

- THERE IS ABSOL. $\bar{a}$ WRT UNIVERSE

(2) CAN'T TELL IF I'M MOVING " "

- LAWS DON'T DEP. ON $\bar{v}$
- ALL FRAMES w/ CONST. $\bar{v}$ REL TO INERTIAL FRAME ARE INERTIAL FRAMES $\Rightarrow$ $\bar{a}$ IS SAME IN ALL
- NO ABS. $\bar{v}$ IN NEWT. MECH WRT UNIV. (ONLY RELATIVE) $\Rightarrow$ NO EXPT GIVES $\bar{v}$

"NEWTONIAN RELATIVITY"

LAW (3)

- WORKS FOR $v \ll c$ = SPD OF LIGHT
- WILL APPLY TO CENTRAL $\bar{F}$'s
- GIVES $\bar{p}$ CONS.
- PROBLEMS:

(a) PROPAGATION TIME (ex GRAVITY, E/M)

$\Rightarrow$ (3) IS INSTANTANEOUS (OK IF OBJECTS SLOW)

(b) MAGN. FIELD:

$\bar{F}$'s FROM CHG PARTICLES PT IN DIFF DIRS.

EFFECT IS SMALL IF $v \ll c$ CP. TO COULOMB $\bar{F}$

SOLVE SOME CASES:

VECTOR EQN $\Rightarrow$ 3 EQNS

$$\sum F_i = m a_i \qquad i = 1, 2, 3$$

(LIKE UNITS: IF LHS IS VECT, SO IS RHS
ELSE WOULD FAIL IN ROTATED
COORD SYS)

$\bar{F}$ CONST: (MOSTLY WHAT CONSIDERED IN INTRO COURSE)

$$\bar{F} = m\bar{a} \Rightarrow \ddot{\bar{r}} = \frac{\bar{F}}{m}$$

SOLVE BY INTEGRATING (DON'T NEED COMPONENTS)

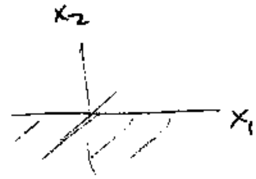
$$\dot{\bar{r}} = \bar{v} = \frac{1}{m}\bar{F}t + \bar{C}_0 \Rightarrow \bar{C}_0 = \bar{v}(0) \equiv \bar{v}_0$$

$$\bar{r} = \frac{1}{2m}\bar{F}t^2 + \bar{v}_0 t + \bar{C}_1 \Rightarrow \bar{C}_1 = \bar{r}(0) = \bar{r}_0$$

$$\Rightarrow \boxed{\begin{aligned} \bar{r} &= \frac{1}{2m}\bar{F}t^2 + \bar{v}_0 t + \bar{r}_0 \\ \bar{v} &= \frac{1}{m}\bar{F}t + \bar{v}_0 \end{aligned}}$$

TRUE COMP. BY COMP.

ex GRAV NEAR EARTH



(ignore x_3)

$$\bar{F}_g = m\bar{g} \equiv -mg\,\hat{e}_2$$

$$\Rightarrow \quad x_1 = v_1^\circ t + x_1^\circ \qquad x_2 = -\tfrac{1}{2} g t^2 + v_2^\circ t + x_2^\circ$$

$$v_1 = v_1^\circ \qquad v_2 = -gt + v_2^\circ$$

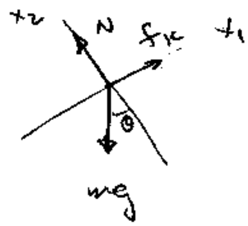
ONLY $\bar{r}_0$, $\bar{v}_0$ DISTINGUISH ONE CASE FROM OTHER (*ex* DROPPING FROM REST VS PROJECTILE)

INTERESTING FEATURES: (of intro problems)

1. FINDING $\Sigma\bar{F}$ $\Rightarrow$ FREE BODY DIAGRAMS } continue to use

2. CHOOSING GOOD COORD SYS

ex



$\Rightarrow$ MOTION IS ONLY IN x_1

ex

WANT t TO LAND

$\Rightarrow$ MOVE W/ $v_1^\circ = v^\circ \cos\theta$

MORE INTERESTING CASES:

RETARDING $\vec{v}$ - DEPENDENT FORCES (WIND/FLUID DRAG)

EXPECT $\quad \bar{F}_r = -c\, v^n\, \hat{v}$ $\quad \left\{ \begin{array}{l} \text{OPP MOTION, INCR. w/ } v \\ \text{NOTE: } \hat{v} = \bar{v}/v \end{array} \right.$

C : DEP. ON ρ (FLUID DENSITY), A (CROSS SECTIONAL AREA)

OTHERS: VISCOSITY, MATERIAL OF OBJECT, FLOW INDUCED, ...

- NONCONSERVATIVE
- COMPLICATED {NET EFFECT OF MOLECULAR BINDING, ... NOT A FUNDAMENTAL FORCE
- TO DO VERY ACCURATELY: MEASURE & SOLVE NUMERICALLY

DATA: (SMALL OBJECT)

v	n
$0 - 24\frac{m}{s}$ $\left(86 \frac{km}{h}\right)$	~1
$24 - 330\, m/s$ $\left(1200 \frac{km}{h}\right)$ $= v_{SOUND} \equiv$ MACH 1	~2
$> v_{SOUND}$	~1 BUT BIG SLOPE

IF RANGE SMALL: MODEL w/ SINGLE TERM, SIMPLE POWER

ex $\quad n = 1$, 1D MOTION, NO OTHER $\bar{F}$:

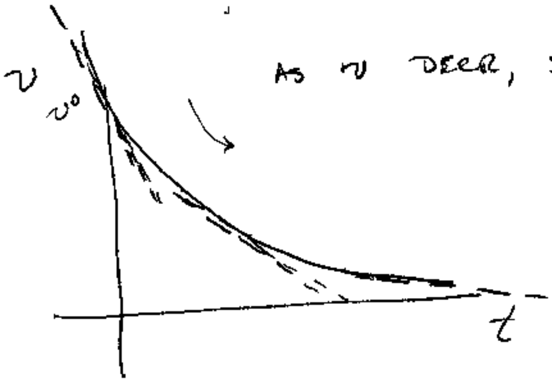
$$ma = F_r$$

$$m\dot{v} = -c\,v$$

$$\boxed{\dot{v} = -\frac{c}{m}\,v}$$

(TEXT: $k \equiv c/m$)
(leave m in since easier to see its role)

- 1^{ST} ORDER DIFF EQ FOR $v(t)$
- CAN'T INTEGRATE AS BEFORE: DON'T KNOW RHS
- SIMPLE TO SOLVE, SKETCH FIRST
  - (a) TAKE $v_i > 0$
  - (b) SLOPE < 0; BIGGER v, GREATER NEG SLOPE, MORE RAPID SLOWDN.
  - (c) AS $v \to 0$, $\dot{v} \to 0$



AS v DECR, SLOPE LESS NEG

(repeat for $v^0 < 0$ )

(d) SIMILAR TO BACTERIA GROWTH, OR COMPOUNDED INTEREST:
RATE $\propto$ AMT (but here rate is negative)

(e) $\Leftarrow$ INSERT

(f) COULD GUESS SOLN & PLUG IN: $v(t) = a e^{-bt}$?

(g) CAN SOLVE EXPLICITLY:

$$\frac{dv}{dt} = -\frac{c}{m} v \qquad \text{USE } dv = \underbrace{\left(\frac{dv}{dt}\right)}_{-\frac{c}{m}v} dt$$

$$\Rightarrow dv = \left(-\frac{c}{m}\right) v\, dt$$

(same as if treated dv/dt as fraction & mult by dT; if thought of as $\frac{\Delta v}{\Delta t}$, really is same)

$$\Rightarrow \int \frac{dv}{v} = -\frac{c}{m} \int dt \qquad \text{div by } v \text{ \& integrate}$$

$$\ln v = -\frac{c}{m} t + d$$

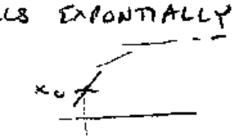
$$v = e^{d} e^{-\frac{c}{m}t} \equiv d' e^{-\frac{c}{m}t}$$

$$v(0) \equiv v_0 = d'$$

$$\Rightarrow \boxed{v(t) = v_0 e^{-\frac{c}{m}t}}$$

(h) GET x:

$$\frac{dx}{dt} = v_0 e^{-\frac{c}{m}t} \qquad \text{\} SLOPE FALLS EXPONENTIALLY}$$



$$x(t) = -\left(\frac{m}{c}\right) v_0 e^{-\frac{c}{m}t} + f$$

$$x_0 = -\left(\frac{m}{c}\right) v_0 + f \quad \Rightarrow \quad f = x_0 + \left(\frac{m}{c}\right) v_0$$

$$\boxed{x(t) = x_0 + \frac{m}{c} v_0 \left[1 - e^{-c/m\, t}\right]}$$

(2ND order eqn $\Rightarrow$ 2 consts)

RANGE: $x(\infty) - x(0) = \frac{m}{c} v(0)$

(could also sketch directly from $\frac{d^2x}{dt^2} = -\frac{c}{m}\frac{dx}{dt}$)

⇒ INSERT:

(e) SIMPLE NUMERICAL SOLUTION (1d)

$$\frac{dv}{dt} = -\frac{c}{m} v$$

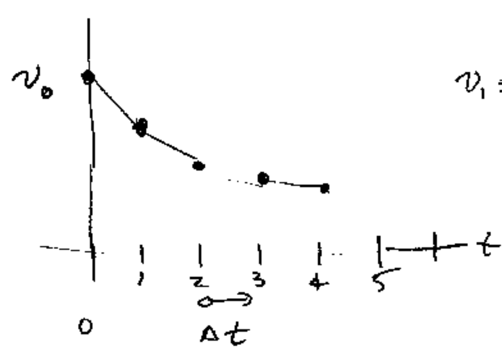
$$\approx \quad \frac{\Delta v}{\Delta t} = -\frac{c}{m} v(t)$$

$$\left.\begin{array}{l} \Delta v = -\frac{c}{m} v(t)\,\Delta t \\ v(t+\Delta t) - v(t) \end{array}\right\} \Rightarrow \boxed{v(t+\Delta t) = v(t) - \frac{c}{m} v(t)\,\Delta t}$$

MORE GENERALLY, FOR $F_r = -f(v)$

$$\dot{v} = -\frac{1}{m} f(v)$$

$$\Rightarrow \boxed{v(t+\Delta t) = v(t) - \frac{1}{m} f(v(t))\,\Delta t}$$



$v(0) = v_0$ (YOU SUPPLY)

$$v_1 = v(\Delta t) = v(0) - \frac{1}{m} f(v(0))\,\Delta t$$

$$= v_0 - \frac{1}{m} f(v_0)$$

$$v_2 = v(2\Delta t) = v_1 - \frac{1}{m} f(v_1)\,\Delta t$$

REPEAT

ie $\boxed{v_n = v(n\Delta t) = v_{n-1} - \frac{1}{m} f(v_{n-1})\,\Delta t}$

- MORE ACCURATE: Δt SMALLER
- EQN SOLVES ITSELF
- CAN SEE WHY NEED 1 CONST v_0 (TO GET STARTED)

(note: this is a very rich field; there are more sophisticated methods that converge to answer more quickly)

FIND $v(x)$ DIRECTLY:

- COULD SOLVE FOR $t(x)$, SUB IN $v(t)$
- CAN REWRITE EQN:

skip

$$\frac{dv}{dt} = -\frac{c}{m} v = \frac{-c}{m} \frac{dx}{dt}$$

$\int_t$ BOTH SIDES: $\boxed{v = -\frac{c}{m} x + d}$

- OR CHAIN RULE:

orig eqn $\dot{v} = -\frac{c}{m} v$

LHS $\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \overset{\text{RHS}}{-\frac{c}{m} \frac{dx}{dt}} \Rightarrow \boxed{\frac{dv}{dx} = -\frac{c}{m}}$

$\Rightarrow \boxed{v = -\frac{c}{m} x + d}$

INT. CONST:

$$v_0 = -\frac{c}{m} x_0 + d \quad \Rightarrow d = v_0 + \frac{c}{m} x_0$$

$$\Rightarrow \boxed{v(x) = v_0 - \frac{c}{m}(x - x_0)}$$

WHERE STOPS?

$$0 = v_0 - \frac{c}{m}(x_{FINAL} - x_0)$$

$$\boxed{x_F = x_0 + \frac{m}{c} v_0}$$

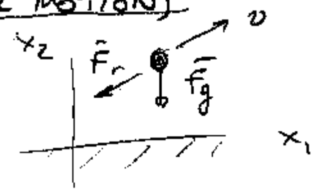
note: ∞ IF $c \to 0$
∞ IF $m \to \infty$

(note that this method gives less info; don't know how long it took, for ex, to get to x_F (∞ t)) *ex* $n=1$ 2D w/ GRAVITY (PROJECTILE MOTION)

$$\bar{F}_r = -c v \hat{v} = -c\bar{v}$$

$$\bar{F}_g = m\bar{g} \equiv -mg\hat{e}_2$$

(note sign)



COMPS:

$$m\bar{a} = m\bar{g} - c\bar{v}$$

(1) $ma_1 = -cv_1$

(2) $ma_2 = -mg - cv_2$

(1) DONE: $m\frac{dv_1}{dt} = -cv_1$

$$\boxed{\begin{aligned} v_1(t) &= v_1^\circ e^{-c/m\, t} \\ x_1(t) &= x_1^\circ + \frac{m}{c} v_1^\circ \left[1 - e^{-c/m\, t}\right] \end{aligned}}$$

(2) $\cancel{m}\frac{dv_2}{dt} = -\cancel{m}g - \frac{c}{m}v_2$

SKETCH: (a) $v_2 > 0 \Rightarrow \frac{dv_2}{dt} < 0 \Rightarrow v_2$ DECR.

(shoot upward)

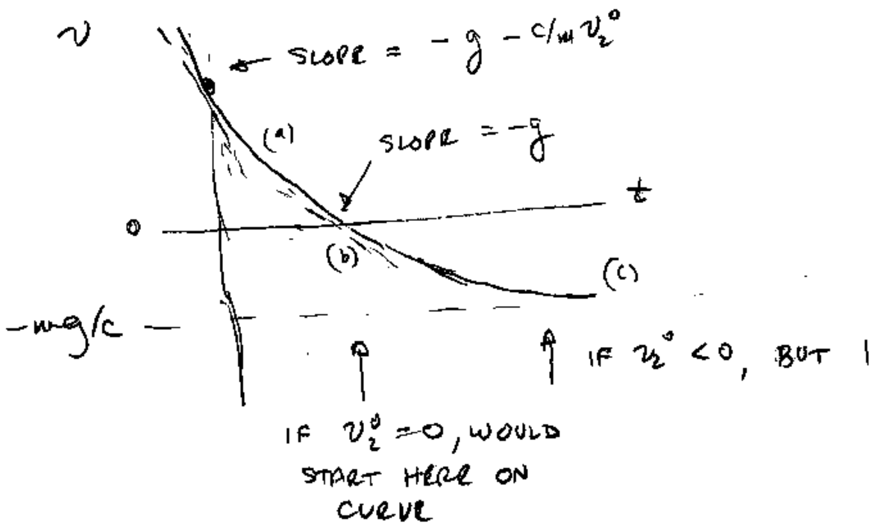
$\frac{dv_2}{dt}$ BECOMES LESS NEG (2ND term RHS)

(b) $v_2 = 0 \Rightarrow \frac{dv_2}{dt} = -g$ (RATHER THAN STOPPING)

(c) $v_2 < 0 \Rightarrow \frac{dv_2}{dt} < 0$ BUT LESS NEG THAN $-g$

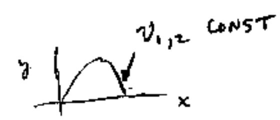
v_2 BECOMES MORE NEG AT SLOWER RATE UNTIL $v_2 =$ CONST

$$0 = -g - \frac{c}{m}v_2 \Rightarrow v_2 = -\frac{mg}{c}$$ "TERMINAL VELOCITY" *



(d) IF $v_2^\circ < -\frac{mg}{c}$

*



so trajectory straightens out

SOLVE: $\displaystyle\int \frac{dv_2}{g + (c/m) v_2} = -\int dt$

$\left\{\right.$ ASIDE: IF $\vec{F}_r = -c v^n \hat{v}$ $\quad \displaystyle\int \frac{dv_2}{g \pm (c/m)|v_2^n|}$

$n=2 \Rightarrow$ TRIG SUBST
IN GENERAL: LOOK UP
SIGN: DEPENDS IF MOVING UP OR DN

$$\frac{m}{c} \ln\left(g + \frac{c}{m} v_2\right) = -t + d$$

SUMMARIZE
(a) SOLVE FOR v_2
(b) FIX CONST BY $v_2(t=0) = v_2^\circ$

$$\ln(\quad) = -\frac{c}{m} t + d'$$

$$g + \frac{c}{m} v_2 = e^{d'} e^{-c/m\, t}$$

$$v_2 = -\frac{mg}{c} + d'' e^{-c/m\, t} \qquad \left\{ d'' = \frac{m}{c} e^{d'} \right.$$

$$v_2^\circ = -\frac{mg}{c} + d'' \Rightarrow d'' = v_2^\circ + \frac{mg}{c}$$

$$\boxed{v_2 = v_2^\circ e^{-c/m t} - \frac{mg}{c}\left[1 - e^{-c/m\, t}\right]}$$

or $\boxed{v_2 = v_2^\circ - \left(\frac{mg}{c} + v_2^\circ\right)\left[1 - e^{-c/m\, t}\right]}$

$$t \to \infty \quad v_2 \to -\frac{mg}{c}$$

GET x_2:

SUMMARIZE
(c) INTEGRATE
(b) SOLVE FOR x_2
(c) FIX CONST BY $x_2(t=0) = x_2^\circ$

$$\frac{dx_2}{dt} = v_2 \equiv A + B e^{-(c/m)t} \qquad \left.\begin{array}{l} A = \frac{mg}{c} \\ B = v_2^\circ + \frac{mg}{c} \end{array}\right.$$

$$\Rightarrow \quad x_2 = At - \frac{m}{c} B e^{-(c/m)t} + d$$

$$x_2^\circ = -\frac{m}{c} B + d \quad \Rightarrow \text{fixes } d$$

PUT TOGETHER

$$\boxed{x_2 = x_2^\circ - \frac{mg}{c} t + \frac{m}{c}\left(v_2^\circ + \frac{mg}{c}\right)\left[1 - e^{-(c/m)t}\right]}$$

skip $\{$ CHALLENGE: SHOW WHEN $c \to 0$, RECOVER SOLNS w/ NO RESIST.

skip $\{$ SMALL t: $\quad x_2 \sim x_2^\circ - \frac{mg}{c} t + \frac{m}{c}\left(v_2^\circ + \frac{mg}{c}\right)\left(1 - \left(1 - \frac{c}{m} t + \cdots\right)\right)$

$$\sim x_2^\circ + v_2^\circ t + \cdots$$

RANGE:

(1) FINAL $t = T$:
WHEN $x_2(T) = 0$

(2) $R = x_1(T)$

$x_1^\circ = x_2^\circ = 0$

$v_1^\circ = v\cos\theta \qquad v_2^\circ = v\sin\theta$

(1) FIND T:

$$x_2(T) = 0 = \cancel{x_2^\circ}^{\,0} - \frac{mg}{c}T + \frac{m}{c}(\underbrace{v_2^\circ}_{v\sin\theta} + \frac{mg}{c})\left[1 - e^{-(c/m)T}\right]$$

ie $$\boxed{T = \left(\frac{v\sin\theta}{g} + \frac{m}{c}\right)\left(1 - e^{-(c/m)T}\right)}$$

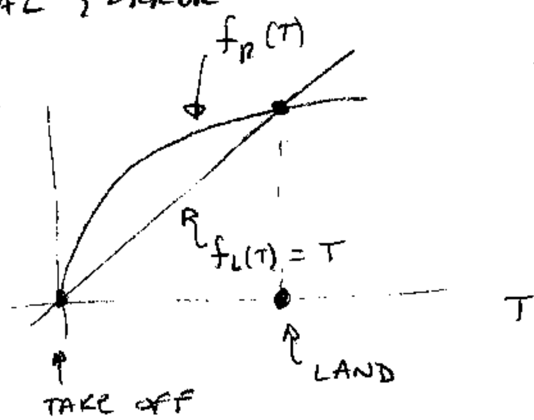
PROBLEM: T ON BOTH SIDES; NO SIMPLE SOLN

METHODS:

(0) TRIAL & ERROR

(1) GRAPH

PLOT LEFT & RIGHT FNS ON SAME GRAPH



f_R : T SMALL $\sim \left(1 + \frac{c\,v\sin\theta}{mg}\right)T$ (SLOPE > 1 if $v\sin\theta > 0$)

T LARGE $\Rightarrow \frac{m}{c}\left(1 + \frac{cv\sin\theta}{mg}\right)$ FLAT

$\Rightarrow$ SECOND SOLN EXISTS IF $v\sin\theta > 0$

{main uses:
- does soln exist?
- estimate where

(2) PERTURBATIVE SOLN:

- USEFUL IF PROBLEM HAS SMALL PARAMETER { extremely useful, eg in QM

"+" HERE: IF WIND RESIST. IS SMALL EFFECT: $k \equiv \frac{c}{m} \sim 0$

FIND $T_{k=0}$ + CORRECTIONS

PERTURBATION FROM $k \neq 0$

PROCEDURE:

(a) EXPAND EQN IN TAYLOR SERIES AROUND $k=0$

(b) WRITE SOLN IN " " FORM: (ie assume could expand in series)

$$T(k) \equiv T^{(0)} + kT^{(1)} + k^2 T^{(2)} + \dots$$

SOLN w/ NO RESIS. — 1^{ST} ORDER CORRECTION — ETC

(c) PLUG IN & SOLVE FOR $T^{(0)}, T^{(1)}, \dots$

MORE TERMS ⇒ MORE ACCURATE

(d) STOP WHEN:

(i) AS ACCURATE AS NEEDED

(ii) PASS OUT

$k \equiv c/m$

$W \equiv \frac{v \sin\theta}{g}$

(a) $$T = \left(W + \frac{1}{k} \right)\left(1 - e^{-kT} \right)$$

LHS: $T^{(0)} + kT^{(1)} + k^2 T^{(2)}$ (WILL KEEP UP TO k^2)

RHS: PROBLEM ALREADY AT 0^{TH} TERM: $\infty \cdot 0$

SOLN: EXPAND PARTS SEPARATELY (GOOD IDEA GENERALLY)

⇒ WILL TELL US HOW FAST 2^{ND} TERM → 0

" " " WHO WINS

USE $e^x \approx 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$ (worth memorizing also $\sin x$, $\cos x$, $\frac{1}{1 \pm x}$)

$$e^{-kT} = e^{-kT^{(0)} - k^2 T^{(1)} - k^3 T^{(2)}}$$

WILL NEED k^3 BECAUSE OF $\frac{1}{k}$ IN 1^{ST} TERM

$$= e^{-kT^{(0)}} e^{-k^2T^{(1)}} e^{-k^3T^{(2)}}$$

$$\sim \left[1 - kT^{(0)} + \tfrac{1}{2}k^2T^{(0)2} - \tfrac{1}{3!}k^3T^{(0)3}\right]\left[1 - k^2T^{(1)} + \tfrac{1}{2}k^4T^{(1)2}\right]\left[1 - k^3T^{(2)}\right]$$

$$= 1 - kT^{(0)} + \ldots$$

THEN

$$1 - e^{-kT} \approx 1 - (\;''\;)$$

$$\approx kT^{(0)} - k^2\left[\tfrac{1}{2}T^{(0)2} - T^{(1)}\right] + k^3\left[\tfrac{1}{3!}T^{(0)3} - T^{(0)}T^{(1)} + T^{(2)}\right]$$

FINALLY

$$T = \underbrace{\left(W + \tfrac{1}{k}\right)}_{\text{DON'T EXPAND; ALREADY AS SIMPLE AS POSSIBLE}}\left(1 - e^{-kT}\right)$$

$$\Rightarrow \quad T^{(0)} + kT^{(1)} + \ldots = \left(W + \tfrac{1}{k}\right)\left(kT^{(0)} - k^2\left[\tfrac{1}{2}T^{(0)2} - T^{(1)}\right] + \ldots\right)$$

NO PROBLEM — ALL AT LEAST $\propto k$

COLLECT IN SINGLE SERIES: (LOTS OF CANCELS)

$$0 = k\underbrace{\left[W\,T^{(0)} - \tfrac{1}{2}T^{(0)2}\right]}_{(1)} + k^2\underbrace{\left[-W\left(\tfrac{1}{2}T^{(0)2} - T^{(1)}\right) + \tfrac{1}{3!}T^{(0)3} - T^{(0)}T^{(1)}\right]}_{(2)}$$

REQUIRE:
EACH TERM VANISHES (SINCE $=0$ FOR ANY k)

(1) $\boxed{T^{(0)} = 2W = \dfrac{2v\sin\theta}{g}}$ $F_r = 0$ RESULT

(2) SUB IN $T^{(0)}$ $\Rightarrow$ $\boxed{T^{(1)} = -\dfrac{2}{3}\left(\dfrac{v\sin\theta}{g}\right)^2}$

i.e.

$$T \approx T^{(0)} + kT^{(1)}$$

$$\boxed{T = \frac{2v\sin\theta}{g} - \left(\frac{c}{m}\right)\frac{2}{3}\left(\frac{v\sin\theta}{g}\right)^2 + \cdots}$$

NOTES:

- ONLY GOOD FOR $\frac{c}{m}$ SMALL ; MORE TERMS THE BETTER
- FAILS ? WANT 1^{ST} CORRECTION $\ll$ 0^{TH}

$$\Rightarrow \quad c \ll 3\frac{mg}{v\sin\theta}$$

FAILS IF $c \approx$ RHS
(ex: m small, g small
v large)

skip
- CAN ORDER IN DIFF WAYS
  - ex TEXT : GET LOWEST TERM
    SUB BACK IN TO GET 1^{ST} CORRECTION
    SUB " " $0^{TH} + 1^{ST}$ TO GET 2^{ND} . . . .

RANGE:

$$R = x_1(T) = \frac{v_1^0}{k}\left[1 - e^{-kT}\right]$$

(a) EXPAND TO ORDER k

(b) KEEP T TO ORDER k

$$\Rightarrow \quad R = v_1^0\left[T^{(0)} + kT^{(1)} - \tfrac{1}{2}kT^{(0)2} + \cdots\right]$$

$$= \underbrace{\frac{2v^2\sin\theta\cos\theta}{g}}_{R^{(0)} = \frac{v^2\sin 2\theta}{g}} - \frac{c}{m}\frac{8}{3}\frac{v^3\sin^2\theta\cos\theta}{g^2} + \cdots$$

$$\boxed{R = R^{(0)}\left[1 - \frac{c}{m}\frac{4}{3}\frac{v\sin\theta}{g} + \cdots\right]}$$

cost of wind resis.
bigger for c larger → not surprise
why for θ larger? g smaller?

(3) ITERATIVE SOLN (FASTEST, MOST FUN)

w/ ONLY $+, -, \times, /$

ex $\sqrt{2}$

USE $x = \frac{2}{x}$ (true when $x = \sqrt{2}$)

STEPS: (1) GUESS x_0

(2) x_1 = AVE OF LHS & RHS

$= \frac{1}{2}\left(x_0 + \frac{2}{x_0}\right)$ } NOTE: IF x_0 IS LOW, $\frac{2}{x_0}$ IS HIGH, AVE IS CLOSER

(3) REPEAT: $\boxed{x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)}$

$\Rightarrow$ 12 DECIMAL PLACES IN 4 ITERATIONS (USING $x_0 = 1$)

OUR PROBLEM $\quad T = \left(W + \frac{1}{k}\right)\left(1 - e^{-kT}\right)$

- EASIER ALGORITHM (The one above works & is faster)

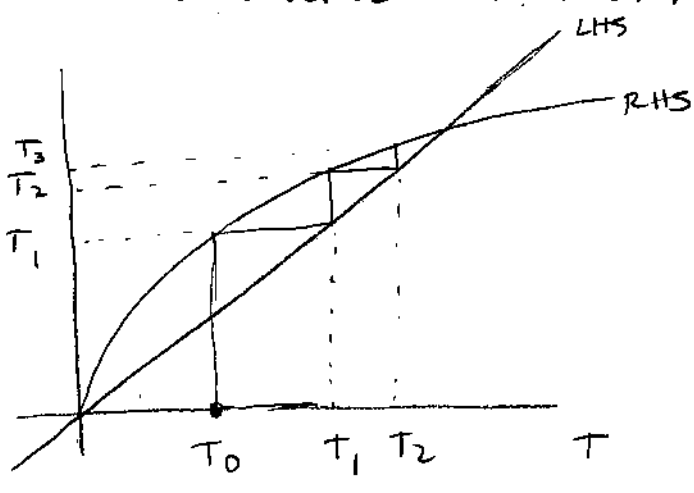
(1) PUT GUESS INTO RHS

(2) USE LHS AS NEXT ESTIMATE

(3) REPEAT

$\Rightarrow T_{n+1} = \left(W + \frac{1}{k}\right)\left(1 - e^{-kT_n}\right)$

- KNOW WILL CONVERGE FROM PLOT:



ex: $T = 2(1 - e^{-T})$

$T_0 = 1$

$T = 1.59362$

AFTER 13 ITS

WORKS FROM EITHER SIDE

- OTHERS ARE POSSIBLE: INVENT YOUR OWN

(4) LET MATHEMATICA, MAPLE, MATLAB, ETC SOLVE IT FOR YOU (CF APPX H)