

1. Read A&H Ch 7

True/False: I read this material.

2. Using the unitary time and coordinate translation operators, show that the vacuum expectation value for any two fields operators  $A(x)$  and  $B(y)$ ,

$$\langle 0| A(x)B(y) |0\rangle$$

is a function only of the difference  $x - y$ , as you would expect for a translationally invariant vacuum. Here  $x$  and  $y$  are the usual position four-vectors. (This is simple - just a few lines.)

3. A&H problem 7.2

4. Show that the time ordering between two timelike separated events is preserved under proper Lorentz transformations. (Proper includes rotations and boosts, but not parity or time reversal.) It may help to first show that  $\Lambda^0_0 \geq 1$ .

5. Construct the rotation  $J^3 \equiv M^{12}$  and boost  $K^3 \equiv M^{03}$  generators for the interacting  $\lambda\phi^4$  theory in terms of fields  $\phi(x)$  and  $\pi(x)$ . You should find that  $K^3$  depends on  $\lambda$ , while  $J^3$  does not. (If expressed in terms of creation and destruction operators, you would find  $K^3$  changes particle number, while  $J^3$  does not.) Explain why  $K^3$  cannot commute with  $H \equiv P^0$ , but is nevertheless conserved (that is, independent of  $t$ ). How can this be consistent with the Heisenberg equations of motion?

6. (Optional) Express  $J^3$  in terms of free-particle creation and destruction operators. Show that  $J^3$  leaves particle number unchanged, and generates the appropriate transformation on a one-particle state  $|\mathbf{p}\rangle$ . For what  $\mathbf{p}$ 's is this an eigenstate? (I decided to make this optional, since it's a bit tedious to work out. But it's worth doing if you're interested.)

7. Consider a theory for two different scalar particles, labelled  $A$  and  $B$ , with  $m_A > 2m_B$ . If the Hamiltonian includes an interaction term

$$H_{ABB} = g \int d^3x \phi_A(x)\phi_B^2(x)$$

compute the matrix element

$$\langle \mathbf{q}_1, \mathbf{q}_2 | H_{ABB} | \mathbf{p} \rangle$$

(in the interaction picture) which appears in the first-order calculation of the decay  $A \rightarrow BB$ . (Recall Fermi's Golden Rule.) Here  $\mathbf{p}$  is the initial  $A$  momentum, and the  $\mathbf{q}$ 's are the final  $B$  momenta.

8. While the interaction picture is simple to work with, the Schrödinger picture is more intuitive. (The operators are what you think they are.) Here you will convert the perturbative expansion for time evolution operator in the interaction picture,  $U_I(t, t_0)$ , to the Schrödinger version.

(a) Show that

$$U_S(t, t_0) = e^{-iH_0 t} U_I(t, t_0) e^{iH_0 t_0}$$

with  $t_0$  at one end, but  $t$  at the other.

(b) Convert the expansion

$$U_I(t, t_0) \approx 1 + (-i) \int_{t_0}^t dt_1 H'_I(t_1) + (-i)^2 \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H'_I(t_1) H'_I(t_2) + \dots$$

into an expansion for  $U_S(t, t_0)$ , expressed in terms of the Schrödinger interaction  $H'(t)$  and the unperturbed evolution operator

$$U_S^0(t_a, t_b) = e^{-iH_0(t_a - t_b)} .$$

(The result is a simple intuitive picture in which the system evolves freely, interacts, evolves freely, interacts again, and so on, summed over all possible times between interactions.)